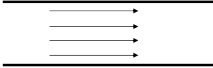


## One Dimensional Flow

- The first flow situation we will investigate is that of One Dimensional, Inviscid, Adiabatic Flow.

- This type flow can be visualized as that through a constant area pipe:



- At first, this case seems trivial since incompressible flow would require that nothing happen.
- This "trivial" solution also occurs in compressible flow, but it is not the only possibility.
- The solutions to this flow will be the building block of for other flow situations.

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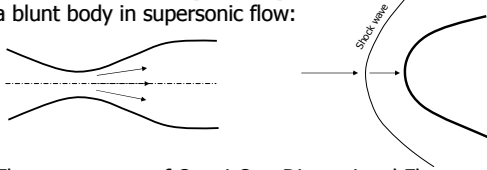
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## One Dimensional Flow [2]

- There are also a number of 'nearly' one dimensional flow situations.
- For example the flow in a converging/diverging duct or the flow along the stagnation streamline of a blunt body in supersonic flow:



- These are cases of Quasi-One Dimensional Flow which will be discussed in later chapters.

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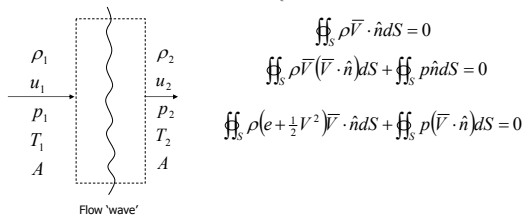
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## 1-D Flow Equations

- For 1-D flow, the velocity reduces to a single component,  $u$ , which we will align with the  $x$  axis.
- We will only consider steady flow, so the mass and momentum conservation equations become:




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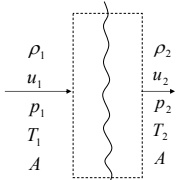
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### 1-D Flow Equations [2]

- For the flow through the control volume shown, we allow for the possibility of a flow disturbance in the form of a wave – either a pressure or shock-wave.

- By integrating over the inflow/outflow boundaries:



Flow 'wave'

$$\begin{aligned} \iint_S \rho \vec{V} \cdot \hat{n} dS &= 0 \\ -\rho_1 u_1 A + \rho_2 u_2 A &= 0 \\ \boxed{\rho_1 u_1 = \rho_2 u_2} \\ \iint_S \rho \vec{V} (\vec{V} \cdot \hat{n}) dS + \iint_S p \hat{n} dS &= 0 \\ -\rho_1 u_1^2 A - p_1 A + \rho_2 u_2^2 A + p_2 A &= 0 \\ \boxed{p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2} \end{aligned}$$

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### 1-D Flow Equations [3]

- And for the energy equation:

$$\begin{aligned} \iint_S \rho (e + \frac{1}{2} V^2) \vec{V} \cdot \hat{n} dS + \iint_S p (\vec{V} \cdot \hat{n}) dS &= 0 \\ -\rho_1 (e_1 + \frac{1}{2} u_1^2) u_1 A - p_1 u_1 A + \rho_2 (e_2 + \frac{1}{2} u_2^2) u_2 A + p_2 u_2 A &= 0 \\ -\rho_1 u_1 \left( e_1 + \frac{p_1}{\rho_1} + \frac{1}{2} u_1^2 \right) + \rho_2 u_2 \left( e_2 + \frac{p_2}{\rho_2} + \frac{1}{2} u_2^2 \right) &= 0 \end{aligned}$$

- But, from the mass conservation (continuity) equation,  $\rho_1 u_1 = \rho_2 u_2$ . Thus:

$$\begin{aligned} e_1 + \frac{p_1}{\rho_1} + \frac{1}{2} u_1^2 &= e_2 + \frac{p_2}{\rho_2} + \frac{1}{2} u_2^2 \\ \boxed{h_1 + \frac{1}{2} u_1^2 = h_2 + \frac{1}{2} u_2^2} \end{aligned}$$

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### 1-D Flow Equations [3]

- If the inflow conditions are known, that leaves us 5 unknowns at the outflow:  $p_2, \rho_2, u_2, T_2, h_2$ .

- So far we only have 3 equations – so we need two more relations to obtain a solution.

- The enthalpy and temperature are, of course, related:

$$\boxed{h_2 = c_p T_2}$$

- This thermally perfect relation adds one equation – and the perfect gas law give us the needed 5<sup>th</sup>.

$$\boxed{p_2 = \rho_2 R T_2}$$

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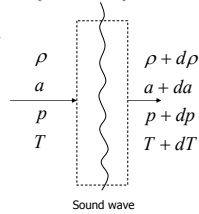
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## Speed of Sound

- A special case of 1-D flow is that of a very weak pressure wave – i.e., a sound wave.
- In this case, put the control volume in motion with the wave so that the inflow velocity is the speed of sound,  $a$ .
- Also allow for the possibility of a change in flow properties across the wave.
- Since a sound wave is weak, express these changes as differential quantities.




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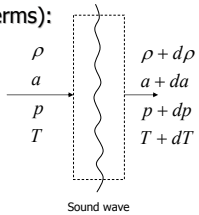
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## Speed of Sound [2]

- In this case, the conservation of mass becomes (after dropping higher order terms):

$$\begin{aligned} \rho a &= (\rho + d\rho)(a + da) \\ &= \rho a + \rho da + a d\rho + d\rho da \\ da &= -a \frac{d\rho}{\rho} \end{aligned}$$



- And momentum becomes:

$$\begin{aligned} p + \rho a^2 &= p + dp + (\rho + d\rho)(a + da)^2 \\ &= p + dp + \rho a^2 + a^2 d\rho + 2\rho a da + \rho da^2 + a d\rho da + d\rho da^2 \\ da &= -\frac{dp + a^2 d\rho}{2\rho a} \end{aligned}$$

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## Speed of Sound [3]

- Combine the two equations by eliminating  $da$ :

$$\begin{aligned} a \frac{d\rho}{\rho} &= \frac{dp + a^2 d\rho}{2\rho a} \\ a &= \frac{1}{2a} \frac{dp}{d\rho} + \frac{a}{2} \\ a^2 &= \frac{dp}{d\rho} \end{aligned}$$

- This last expressions is a differentiation and to be precise, it should be a partial differentiation with one other property held constant.
- Since the flow is adiabatic and inviscid, it is natural to require isentropic (constant entropy) flow.

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### Speed of Sound [4]

- Thus, the speed of sound can be written as:

$$a^2 = \left( \frac{\partial p}{\partial \rho} \right)_s$$

- Also note, that given our previous definition of the compressibility factor, the speed of sound can also be written as:

$$a^2 = \left( \frac{\partial p}{\partial \rho} \right)_s = -v^2 \left( \frac{\partial p}{\partial v} \right)_s = \frac{v}{\tau_s}$$

$$a = \sqrt{\frac{1}{\rho \tau_s}}$$

- Thus we see the close relationship between compressibility and the speed of sound.

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### Speed of Sound [5]

- While the previous equations are interesting in understanding flow behavior, they don't help much in actual calculations.

- To obtain a useful equation, apply our isentropic relation:

$$\left( \frac{p_2}{p_1} \right) = \left( \frac{\rho_2}{\rho_1} \right)^\gamma \Rightarrow \frac{p_2}{\rho_2^\gamma} = \frac{p_1}{\rho_1^\gamma}$$

- If the grouping of properties at the two locations can be separated, they must separately equal a constant. Thus:

$$p = C \rho^\gamma$$

- As it turns out, we don't really need to know the value of the constant,  $C$ .

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### Speed of Sound [6]

- Instead, it can be eliminated when we perform the differentiation:

$$\frac{\partial p}{\partial \rho} = \frac{\partial}{\partial \rho} (C \rho^\gamma) = \gamma C \rho^{\gamma-1} = \frac{\gamma C \rho^\gamma}{\rho} = \frac{\gamma p}{\rho}$$

- And thus,

$$a = \sqrt{\frac{\gamma p}{\rho}}$$

- Also, the perfect gas law can be used to obtain:

$$a = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\gamma R T}$$

- Note this dependence on temperature, and thus the speed of the random motion of the particles, also makes good sense.

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### Forms of the Energy Equation

- Before going on, it is important to spend a little time considering different forms of the energy eqn.

$$h_1 + \frac{1}{2}u_1^2 = h_2 + \frac{1}{2}u_2^2$$

- As before, if the properties at two locations can be separated, they must each equal a constant.
- For this case, we will give the constant a name – the total enthalpy.  $h + \frac{1}{2}u^2 = h_o$
- We indicate this 'total' property with a subscript zero since it is also the value at zero velocity.
- From incompressible flow, we might also call this the 'stagnation' enthalpy.

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### Forms of the Energy Equation [2]

- We can also use the relationship between enthalpy and temperature to write either of:

$$c_p T_1 + \frac{1}{2}u_1^2 = c_p T_2 + \frac{1}{2}u_2^2 \quad c_p T + \frac{1}{2}u^2 = c_p T_o$$

- Further, if the relation between the speed of sound and temperature is introduced...

$$c_p T = \frac{\gamma R T}{\gamma - 1} = \frac{a^2}{\gamma - 1}$$

- Then, we get:

$$\frac{a_1^2}{\gamma - 1} + \frac{u_1^2}{2} = \frac{a_2^2}{\gamma - 1} + \frac{u_2^2}{2} \quad \frac{a^2}{\gamma - 1} + \frac{u^2}{2} = \frac{a_o^2}{\gamma - 1}$$

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### Forms of the Energy Equation [3]

- All the previous equations are valid forms of the adiabatic energy equation.
- One form relates the properties at two points in a flow to each other while the other form relates the properties at any point to the reference, total conditions.
- This is useful since, for adiabatic flow, the total flow conditions of  $h_o$ ,  $T_o$ , and  $a_o$  do not change!
- We will later see that these 1-D equations are also valid in 2 and 3-D if the velocity is replaced with the total velocity magnitude:  $u \rightarrow V$

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### Forms of the Energy Equation [4]

- For external flows, the total or stagnation conditions are the preferred reference values.
- In internal flows, like engines, there is another set of reference conditions often used. These are the **sonic conditions** – or those that would occur at the speed of sound.
- Using an asterisk to denote sonic conditions, one form of the energy equation is:

$$\frac{a^2}{\gamma-1} + \frac{u^2}{2} = \frac{a^{*2}}{\gamma-1} + \frac{u^{*2}}{2} = \frac{\gamma+1}{2(\gamma-1)} a^{*2}$$

- Note that by definition:  $u^* = a^*$

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### Forms of the Energy Equation [5]

- Similarly, the sonic temperature can be introduced:

$$c_p T + \frac{1}{2} u^2 = c_p T^* + \frac{1}{2} a^{*2}$$

- Note also that the sonic and total conditions can be related:

$$\frac{a_0^2}{\gamma-1} = \frac{a^2}{\gamma-1} + \frac{u^2}{2} = \frac{\gamma+1}{2(\gamma-1)} a^{*2}$$

- Thus:

$$\frac{a^{*2}}{a_0^2} = \frac{T^*}{T_0} = \frac{h^*}{h_0} = \frac{2}{\gamma+1}$$

- It follows that these sonic conditions, like the total conditions are flow constants.

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### Mach Relation

- Some final, and probably the most useful, forms of the energy equation involve the Mach number.

$$\frac{a^2}{(\gamma-1)} + \frac{1}{2} V^2 = \frac{a_0^2}{(\gamma-1)}$$

- Rearrange to get the ratio of total to local temperature on one side:

$$\frac{a_0^2}{a^2} = \frac{T_0}{T} = 1 + \frac{(\gamma-1)}{2} \frac{V^2}{a^2}$$

- Now, introduce the Mach number to get the first of our "Mach Relation" equations:

$$\frac{T_0}{T} = 1 + \frac{(\gamma-1)}{2} M^2$$

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### Limits of Adiabatic Flow Assumption

- All of the equations to this point are valid for any adiabatic flow – which is pretty much all aerodynamic flow cases.
- However, there are some important situations where the above equation doesn't work:
  - Obviously, whenever there is heat addition – the most common of which is actively cooled hypersonic and space reentry flows.
  - Whenever there is a propeller, compressor, or turbine.
  - Two merging flows from separate sources.
- In these cases, the equations don't work because the total enthalpy is not a constant – and thus neither is the total temperature.

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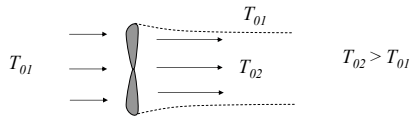
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### Limits of Adiabatic Flow Assumption [2]

- This brings up an important point – the total (and sonic) conditions are reference conditions, they don't necessarily correspond to a point in the flow.
- However, all points in a flow have a total and sonic temperature associated with them – these are a measure of the energy at the point.
- In the cases mentioned, the energy (internal plus kinetic) is not a constant throughout:



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### Isentropic Flow Relations

- While the previous equations are good for any adiabatic flow, there are also many cases when the flow is also reversible – and thus isentropic.
- From our previous isentropic flow relations:
$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\gamma-1} \quad \frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma-1}{\gamma}}$$
- These equations relate the properties at one locations to that of another – as long as the flow between the two points is isentropic!
- Thus, this equation will not work in a viscous boundary layer or across a shock wave.

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### Isentropic Flow Relations [2]

- We can use these equations to also calculate the total pressure and total density:

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}} \quad \frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{\frac{1}{\gamma-1}}$$

- As with the total enthalpy and temperature, these reference quantities don't have to be actual points in the flow.
- Thus, the total pressure is the pressure the flow would have **IF** it were isentropic brought to rest.
- Similarly, the total density is the density the flow would have **IF** isentropically brought to rest.

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### Isentropic Flow Relations [3]

- Using these relations, we can then write:

$$\frac{p_0}{p} = \left(1 + \frac{(\gamma-1)}{2} M^2\right)^{\frac{\gamma}{\gamma-1}} \quad \frac{\rho_0}{\rho} = \left(1 + \frac{(\gamma-1)}{2} M^2\right)^{\frac{1}{\gamma-1}}$$

- And the sonic pressure and temperature can be found from:

$$\frac{p^*}{p_0} = \left(\frac{T^*}{T_0}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho^*}{\rho_0} = \left(\frac{T^*}{T_0}\right)^{\frac{1}{\gamma-1}} = \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}$$

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### Dynamic Pressure

- When non-dimensionalizing forces and pressures in compressible flow, it is still convenient to use the dynamic pressure. I.e.:

$$q = \frac{1}{2} \rho V^2 \quad C_L = \frac{L}{qS} \quad c_p = \frac{(p - p_\infty)}{q}$$

- However, remember that Bernoulli's equation **does not** apply in compressible flow!
- To reinforce this, we can rewrite the dynamic pressure in terms of pressure and Mach number:

$$q = \frac{1}{2} \rho V^2 = \frac{1}{2} \frac{\gamma p}{\gamma p} \rho V^2 = \frac{\gamma}{2} \frac{p V^2}{a^2} = \frac{\gamma}{2} p M^2$$

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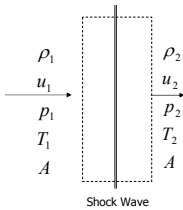
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## Normal Shock Relations

- Finally, let's return to our original problem and look at the case when a shock wave is present.



- In particular, this is called a normal shock because it is perpendicular to the flow.
- The conservations equations in 1-D are still:

$$\begin{aligned}\rho_1 u_1 &= \rho_2 u_2 \\ p_1 + \rho_1 u_1^2 &= p_2 + \rho_2 u_2^2 \\ h_1 + \frac{1}{2} u_1^2 &= h_2 + \frac{1}{2} u_2^2\end{aligned}$$

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## Normal Shock Relations [2]

- Let's start by dividing the two sides of the momentum by the mass conservation equation:

$$\frac{p_1}{\rho_1 u_1} + u_1 = \frac{p_2}{\rho_2 u_2} + u_2$$

- And, by rearranging and introducing the speed of sound:

$$u_2 - u_1 = \frac{p_2}{\rho_2 u_2} - \frac{p_1}{\rho_1 u_1} = \frac{a_2^2}{\gamma u_2} - \frac{a_1^2}{\gamma u_1}$$

- But, from one form of our energy equation:

$$\frac{a^2}{\gamma - 1} + \frac{u^2}{2} = \frac{\gamma + 1}{2(\gamma - 1)} a^{*2}$$

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## Normal Shock Relations [3]

- Or, when written for the two points involved:

$$a_1^2 = \frac{\gamma + 1}{2} a^{*2} - \frac{\gamma - 1}{2} u_1^2 \quad a_2^2 = \frac{\gamma + 1}{2} a^{*2} - \frac{\gamma - 1}{2} u_2^2$$

- Note that since the flow is adiabatic, the sonic speed of sound,  $a^*$ , is the same at both points.
- Substituting these two equations into our previous equation – and rearranging – gives:

$$u_2 - u_1 = \frac{\gamma + 1}{2\gamma u_1 u_2} a^{*2} (u_2 - u_1) + \frac{\gamma - 1}{2\gamma} (u_2 - u_1)$$

- Which looks complex - until you notice the common factor  $(u_2 - u_1)$ .

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### Normal Shock Relations [4]

- This equation is automatically satisfied if nothing happens in the control volume, i.e.  $u_2 = u_1$ .
- This is the trivial case, but it is nice to know our equations will give that result.
- The more interesting case is when  $(u_2 - u_1) \neq 0$ , which allows us to divide through by this factor.

$$1 = \frac{\gamma + 1}{2\gamma u_1 u_2} a^{*2} + \frac{\gamma - 1}{2\gamma}$$

- Or, when rearranged, simply:

$$a^{*2} = u_1 u_2$$

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### Characteristic Mach Number

- Another way of writing this result is in terms of the characteristic Mach number:  $M^* = u/a^*$ .

$$M_1^* = 1/M_2^*$$

- Note that this is not a "true" Mach number which is the ratio of local velocity to local speed of sound.
- This relationship tells us something very important
  - If the flow is initially subsonic,  $u_1 < a^*$ , then it will become supersonic  $u_2 > a^*$ .
  - Of, if the flow is initially supersonic,  $u_1 > a^*$ , then it will become subsonic,  $u_2 < a^*$ .

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### Characteristic Mach Number [2]

- The first possibility, a flow spontaneously jumping from subsonic to supersonic, isn't physically possible - we will show this in a little bit.
- The second case, jumping from supersonic to subsonic is exactly what a normal shock does.
- Why? Usually because there is some disturbance or condition downstream which the flow cannot negotiate supersonically. I.e.:
  - When there is a blunt body the flow must go around
  - When a nozzle has an exit pressure condition which requires subsonic flow

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### Characteristic Mach Number [3]

- The previous characteristic Mach relation, while informative, is not very useful in application.
- Instead, relate the characteristic Mach to the true Mach number by using:

$$\frac{a^2}{\gamma-1} + \frac{u^2}{2} = \frac{\gamma+1}{2(\gamma-1)} a^{*2} \Rightarrow \frac{1}{\gamma-1} \frac{a^2}{u^2} + \frac{1}{2} = \frac{\gamma+1}{2(\gamma-1)} \frac{a^{*2}}{u^2}$$

- When simplified, this becomes:

$$M^{*2} = \frac{(\gamma+1)M^2}{2+(\gamma-1)M^2}$$

- Thus the two values are (relatively) simply related.

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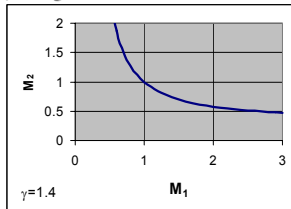
### Mach Jump Relation

- Substituting into our characteristic Mach relation:

$$\frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2} = \frac{2+(\gamma-1)M_2^2}{(\gamma+1)M_2^2}$$

- Or, when simplified, we get the useful relation:

$$M_2^2 = \frac{1 + \frac{(\gamma-1)}{2} M_1^2}{\gamma M_1^2 - \frac{(\gamma-1)}{2}}$$




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### Mach Jump Relation [2]

- It is important to note the limits of the expression as  $M_1 \rightarrow 1$  and  $M_1 \rightarrow \infty$ .

$$\lim_{M_1 \rightarrow 1} (M_2) = \sqrt{\frac{1 + \frac{(\gamma-1)}{2}}{\gamma - \frac{(\gamma-1)}{2}}} = 1$$

$$\lim_{M_1 \rightarrow \infty} (M_2) = \sqrt{\frac{\gamma-1}{2\gamma}} = 0.378 \text{ (in air)}$$

- Thus, if we are sonic, the normal shock becomes very weak and nothing happens.
- If we go hyper-hypersonic, the flow reaches a fixed post-shock Mach number.

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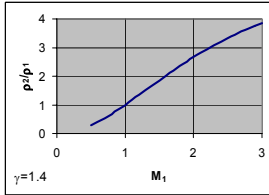
## Density/Velocity Jump Relation

- To get the shock jump relations for our remaining flow properties, start with continuity:

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{u_1^2}{u_1 u_2} = \frac{u_1^2}{a_2^2} = M_1^2$$

- Thus, the density and velocity jumps are inversely related and given by:

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{(\gamma + 1)M_1^2}{2 + (\gamma - 1)M_1^2}$$




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## Density/Velocity Jump Relation [2]

- Once again, if  $M_1=1$ , the shock wave becomes very weak and nothing happens.
- At very high speeds, however:

$$\lim_{M_1 \rightarrow \infty} \frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)}{(\gamma - 1)} = 6 \quad \text{for air}$$

- Thus, when you hear some people talk about hypersonic vehicles compressing air to the density of steel...
- Well, not quite. Not even close actually. But it sure sounds impressive.

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## Pressure Jump Relation

- Next, turn to momentum conservation to get a relation for pressure. First rearrange terms:

$$p_2 - p_1 = \rho_1 u_1^2 - \rho_2 u_2^2 = \rho_1 u_1 (u_1 - u_2)$$

- And then manipulate to get Mach numbers and our previous velocity jump expression:

$$\frac{p_2 - p_1}{p_1} = \frac{\rho_1 u_1^2}{p_1} \left(1 - \frac{u_2}{u_1}\right) \Rightarrow \frac{p_2}{p_1} - 1 = \frac{\gamma M_1^2}{a_1^2} \left(1 - \frac{u_2}{u_1}\right)$$

- Or, just

$$\frac{p_2}{p_1} = 1 + \gamma M_1^2 \left(1 - \frac{1}{M_1^2}\right)$$

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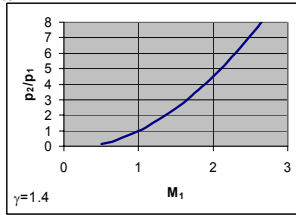
### Pressure Jump Relation [2]

- Finally, insert our definition for characteristic Mach number...

$$\frac{p_2}{p_1} = 1 + \gamma M_1^2 \left( 1 - \frac{2 + (\gamma - 1)M_1^2}{(\gamma + 1)M_1^2} \right)$$

- And, on simplification:

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma + 1} (M_1^2 - 1)$$




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### Pressure Jump Relation [3]

- Once again if  $M_1=1$ , nothing happens.
- Note that this time however, as  $M_1 \rightarrow \infty$ , the pressure also does:  

$$\lim_{M_1 \rightarrow \infty} \frac{p_2}{p_1} = \infty$$
- Thus, while the density might not be huge, the pressures can be.
- Finally, the easiest way to get the temperature jump is through the perfect gas law:

$$\frac{T_2}{T_1} = \frac{p_2 \cdot \rho_1}{p_1 \cdot \rho_2}$$

- So, temperatures also get very large!

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### Entropy Change

- And last, consider the change in entropy across a normal shock wave.
- Using our previous definition and the perfect gas law:

$$s_2 - s_1 = c_p \ln \left( \frac{T_2}{T_1} \right) - R \ln \left( \frac{p_2}{p_1} \right) = c_p \ln \left( \frac{p_2 \cdot \rho_1}{p_1 \cdot \rho_2} \right) - R \ln \left( \frac{p_2}{p_1} \right)$$

- Or with a little extra manipulation:

$$s_2 - s_1 = c_p \ln \left( \frac{\rho_1}{\rho_2} \right) + (c_p - R) \ln \left( \frac{p_2}{p_1} \right)$$

$$s_2 - s_1 = c_p \ln \left( \frac{\rho_1}{\rho_2} \right) + c_v \ln \left( \frac{p_2}{p_1} \right)$$

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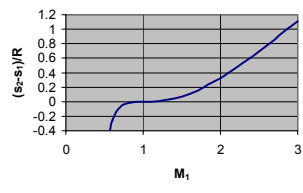
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## Entropy Change [2]

- Now, insert our shock jump relations:

$$s_2 - s_1 = c_p \ln \left( \frac{2 + (\gamma - 1)M_1^2}{(\gamma + 1)M_1^2} \right) + c_v \ln \left( 1 + \frac{2\gamma}{\gamma + 1} (M_1^2 - 1) \right)$$



- Now we see that a subsonic shock,  $M_1 < 1$  would produce a decrease in entropy – something not allowed by the 2<sup>nd</sup> Law of Thermodynamics.
- Thus only supersonic shocks are possible.

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## Total Pressure Jump

- One final thing to note is this special case where a flow is:

- isentropically accelerated from rest to  $M_1$
- jumps through a shock
- and then isentropically slows back down to rest.

- The only entropy change occurs at the shock, thus, we can write for the initial and final states:

$$s_2 - s_1 = c_p \ln \left( \frac{T_{02}}{T_{01}} \right) - R \ln \left( \frac{p_{02}}{p_{01}} \right)$$

- Or, since the flow is adiabatic,  $T_{01} = T_{02}$ . Thus:

$$s_2 - s_1 = -R \ln \left( \frac{p_{02}}{p_{01}} \right)$$

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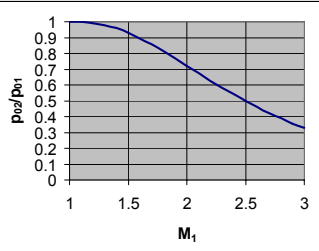
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## Total Pressure Jump [2]

- This can be rewritten as:

$$\frac{p_{02}}{p_{01}} = \frac{p_{02}}{p_{01}} = e^{-\frac{(s_2 - s_1)}{R}}$$

- Thus we see the close relationship between entropy changes and total pressure loss.



- As a result, flow efficiency in inlets and nozzles is often measured by this total pressure ratio.

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