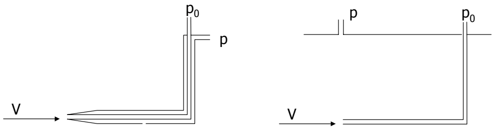
Performance

- Performance is the study of how high, how fast, how far, and how long an aircraft can fly.
- It is one part of the general study of flight dynamics which also include stability and control – however, performance estimates are often made by aerodynamicists.
- In this study, we no longer consider the motion and properties of the air, but now concentrate on the motion of the entire airplane and its response to applied forces.
- One first step is to clarify the different methods of defining aircraft speed.

Airspeed Measurement

- The Pitot-Static system is the standard device for airspeed measurement



- At low speeds, this system makes use of Bernoulli's equation to obtain V from pressures and density

$$p + \tfrac{1}{2}\rho V^2 = p_0 \qquad \Rightarrow \qquad V = \sqrt{\frac{2(p_0 - p)}{\rho}}$$

Airspeed Measurement (continued)

- To measure the aircraft's **True Airspeed**, TAS, at **incompressible** velocities:

$$\boxed{V_t = V_\infty = \sqrt{\frac{2(p_{0\infty} - p_\infty)}{\rho_\infty}}}$$

- However, there is no simple device for measuring density. Thus, airplane instruments are calibrated assuming sea level density, $\rho_{s.}$
- The resulting velocity is called the **Equivalent Airspeed**, EAS,

$$\boxed{V_e = \sqrt{\frac{2(p_{0\infty} - p_\infty)}{\rho_s}}}$$

Airspeed Measurement (continued)

- Notice that EAS and TAS are related by the density ratio, σ,

$$V_e = V_t \sqrt{\frac{\rho_\infty}{\rho_s}} = V_t \sqrt{\sigma}$$

- In fact, EAS is more useful to pilots since equivalent stall speed, $V_{e\ stall}$, is independent of altitude while true stall speed, $V_{true\ stall}$, is not!!
- This is because aerodynamic forces are proportional to dynamic pressure not velocity. At the same V_e you have the same q, at any altitude!

$$q = \tfrac{1}{2}\rho V_t^2 = \tfrac{1}{2}\rho_s V_e^2$$

Airspeed Measurement (continued)

- At subsonic compressible velocities, the true airspeed can be calculated from the isentropic Mach relation (which we will derive later):

$$\frac{p_o}{p} = \left(1 + \tfrac{\gamma-1}{2} M^2\right)^{\gamma/\gamma-1} \qquad \begin{array}{l} \gamma \equiv \text{ratio of specific heats} = 1.4 \text{ for air} \\ \text{M} \equiv \text{Mach Number} \end{array}$$

- From this, the true velocity can be found from:

$$\boxed{V_t = \sqrt{\frac{2\gamma p_\infty}{(\gamma-1)\rho_\infty}\left[\left(\frac{p_{o\infty} - p_\infty}{p_\infty} + 1\right)^{\gamma-1/\gamma} - 1\right]}}$$

- The terms were rearranged since a Pitot-static system measure pressure differences!

Airspeed Measurement (continued)

- In application, aircraft instruments are calibrated assuming sea level air density and pressure, ρ_s and p_s. Thus, the **Calibrated Airspeed**, CAS, is:

$$\boxed{V_c = \sqrt{\frac{2\gamma p_s}{(\gamma-1)\rho_s}\left[\left(\frac{p_{o\infty} - p_\infty}{p_s} + 1\right)^{\gamma-1/\gamma} - 1\right]}}$$

- Believe it or not, this relation reduces to our EAS relation at low velocities, or really low Mach numbers.
- Thus CAS also has the benefit of providing a stall speed which is independent of altitude.

Airspeed Measurement (continued)

- The book notes that the main difference between calibrated and equivalent airspeeds is the assumption of constant density.
- As a result, equivalent airspeed at compressible speeds may be calculated by:

$$\boxed{V_e = \sqrt{\frac{2\gamma p_\infty}{(\gamma-1)\rho_s}\left[\left(\frac{p_{o\infty}-p_\infty}{p_\infty}+1\right)^{\gamma-1/\gamma}-1\right]}}$$

- The factor which relates V_c and V_e is given the symbol *f*, but it is a bit long expression – see the book for the equation and tables for it value. $V_e = f\,V_c$

Airspeed Measurement (continued)

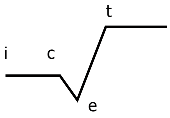
- It has been assumed thus far that the Pitot-static system correctly reads both the total and static pressure and that the instrument displays the right value to the pilot.
- In practice, this is not always true. As a result, even after calibration, there may be sensor position errors in the measure airspeed.
- Thus, the **Indicated Airspeed**, IAS, which is displayed on the cockpit instrument may differ from both EAS and CAS.

$$\boxed{V_c = V_i + \Delta V_p}$$

- ΔV_p is the position errors of the system.

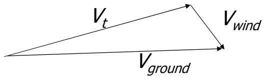
Airspeed Measurement (continued)

- Thus, the process of going from the airspeed a pilot sees to the true air speed is:
  - Convert indicated to calibrated: $V_c = V_i + \Delta V_p$
  - Convert calibrated to equivalent: $V_e = fV_c$
  - Convert equivalent to true: $V_e = V_t\sqrt{\frac{\rho_\infty}{\rho_s}} = V_t\sqrt{\sigma}$
- Because of the sequence of steps and the relative magnitudes of the results, the mnemonic ice-t along with a square root radical is used.

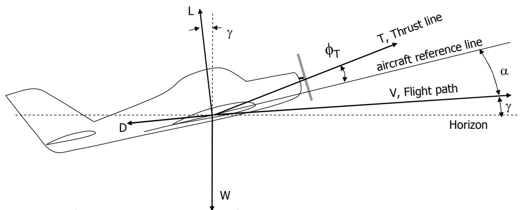


Airspeed Measurement (continued)

- Two final, but important, notes:
- First is that the aviation business still uses knots as the standard unit of airspeed – not ft/sec or m/sec.
- Thus, airspeeds are usually given as KIAS, KCAS, KEAS or KTAS on instruments and in flight manuals.
- And last, the air we fly in is usually not at rest. Thus, Ground Speed of an aircraft is obtained from the vector sum of the airspeed and wind velocities:



Performance (continued)



- Note the new vector angles:
  - Flight path angle, γ : the angle between the velocity vector of the aircraft and the horizon.
  - Thrust line angle, ϕ_T: the angle between the aircraft reference line and the action line of the powerplant.

Performance (continued)

- In your aerodynamics courses you learn how to accurately calculate the lift and drag of aircraft.
- However, in performance, we just need quick estimate, primarily for how drag depends upon lift. For this we use:

$$C_D = C_{D,0} + \frac{C_L^2}{\pi e AR} \qquad e \equiv \text{Oswald efficiency}$$

- The zero lift drag, $C_{D,0}$, is due to viscous effects over the entire airplane surface - wing, fuselage, etc. when C_L=0.
- The second term includes both the span efficiency of the wing and any variation in viscous drag due to lift.

Equations of Motion

- An airplane in flight obeys Newton's Laws of motion. In particular: force = mass * acceleration.
- For airplanes, we split the forces in to those in the flight direction and those perpendicular to it:

$$\sum F_{\parallel} = ma = m\frac{dV}{dt} \qquad \sum F_{\perp} = m\frac{V^2}{r_c}$$

- Note that in the perpendicular equation we allow for a curved flight path with radius r_c.
- Summing forces gives:

$$\boxed{T\cos(\phi_t + \alpha) - D - W\sin\gamma = m\frac{dV}{dt}} \qquad \boxed{L + T\sin(\phi_t + \alpha) - W\cos\gamma = m\frac{V^2}{r_c}}$$

Equations of Motion (continue)

- The previous equations are the general equations of motion for an airplane. They are applicable to all flight conditions.
- A tremendous simplification occurs if we limit the study to steady, level, unaccelerated flight (SLUF).

$dV/dt = 0$ $\qquad r_c \rightarrow \infty$ $\qquad \gamma = 0$

- Also, in most airplanes, the thrust angle is small enough to assume $\cos(\phi_T+\alpha)\sim 1$ and $\sin(\phi_T+\alpha)\sim 0$.
- Under these assumptions,

$$\boxed{T = D \quad \text{and} \quad L = W}$$

Thrust Required

- The thrust acting on an airplane should be considered from two different viewpoints:
  - The thrust required by the airplane to stay in flight at the existing flight conditions, I.e. V, h, γ, etc.
  - The thrust available from the powerplant to maintain or change those flight conditions.
- Lets start with the thrust required. From the previous relations:

$$\boxed{T_R = D = q_\infty S C_D}$$

- Or, since $\dfrac{T = D}{L = W}$

$$\boxed{T_R = \frac{W}{L/D}}$$

Steady,
Level,
Unaccelerated
Flight

Thrust Required (continued)

- The second relation points out a very important point: the minimum thrust require occurs when the airplane lift to drag ratio, L/D = C_L/C_D, is maximum.
- The first equation is more useful however in finding when this occurs. Substituting our previous relation for drag yields:

$$T_R = q_\infty S\left(C_{D,0} + \frac{C_L^2}{\pi e AR}\right) = \underbrace{q_\infty S C_{D,0}}_{\text{Profile or Parasitic Drag}} + \underbrace{\frac{W^2}{q_\infty S \pi e AR}}_{\text{Drag due to lift (Induced drag)}}$$

Thrust Required (continued)

- This equation assumes that L=W as is appropriate for SLUF.
- However, we can include accelerated flight by simply including a load factor, n, term: L = nW.
- In this case:

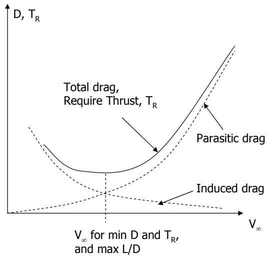
$$\boxed{T_R = \tfrac{1}{2}\rho_\infty V_\infty^2 S C_{D,0} + \frac{2K(nW)^2}{\rho_\infty V_\infty^2 S}}$$

- Where the dynamic pressure term has been expanded, and the symbol K is used to represent:

$$K = \frac{1}{\pi e AR}$$

Thrust Required (continued)

- Note how the two contributions to drag vary differently with velocity:



Thrust Required (continued)

- From this we see that a minimum in required thrust occurs at some value of velocity (or, similarly, q).
- To find this minimum, we differentiate this relation with respect to q_∞ and set the derivative to zero:

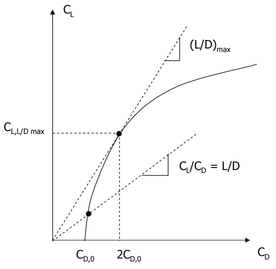
$$\frac{dT_R}{dq_\infty} = \frac{d}{dq}\left\{q_\infty S C_{D,0} + \frac{W^2}{q_\infty S \pi e AR}\right\} = S C_{D,0} - \frac{W^2}{q_\infty^2 S \pi e AR}$$

$$\left.\frac{dT_R}{dq_\infty}\right|_{T_{R,\min}} = 0 \quad \Rightarrow \quad \boxed{C_{D,0} = \frac{W^2}{q_\infty^2 S^2 \pi e AR} = \frac{C_L^2}{\pi e AR}}$$

- Thus, the minimum drag occurs when the parasitic drag and drag due to lift are equal!

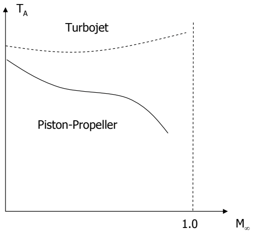
Thrust Required (continued)

- This effect can also be seen by looking at a parabolic drag polar
- Any line from the origin has a slope equal to the L/D ratio.
- Thus, the maximum L/D occurs at the tangency point shown.



Thrust Available

- Thrust available is a function of the power plant type/size and aircraft velocity and altitude.
- Typical thrust available variation with velocity is shown here for two engine types:
- For piston-propeller combinations, thrust decreases at high speed due to Mach effects on the propeller tip.



Thrust Available

- For turbojet engines, thrust normally increases slightly with speed due to the increased inlet performance and increased mass flow rate with Mach number.
- Other engine types like turboprops and turbofans have thrust variations somewhere between these two.
- The best source for engine performance data is the manufacturer themselves provided in the form of an “engine deck”.
- Also realize that engine thrust also depends upon the throttle setting.

Thrust Available (continued)

- For a given airplane, the range of possible steady flight velocities depends upon the relative values of thrust required and thrust available:
- Steady level, un-accelerated flight is only possible when $T_A \geq T_R$.
- To fly at velocities between $V_{\infty,min}$ and $V_{\infty,max}$, the throttle setting would be set less that 100%.