Power Required

- As important as thrust, is power.
- This is particularly true for propeller driven airplanes as evidenced by the fact that piston and turboprop engines are rated in horsepower.
- Fortunately, power and thrust are closely related:

$$\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{\text{Force}\cdot\text{Distance}}{\text{Time}} = \text{Force}\cdot\text{Velocity}$$

- Or, more mathematically:

$$P_R = \vec{T_R}\cdot\vec{V_\infty} = T_R V_\infty$$

For steady level, un-accelerated flight

Power Required (continued)

- For thrust required, we showed that:

$$T_R = \frac{W}{C_L/C_D}$$

- To get a similar relation for power required, remember that:

$$L = W = \tfrac{1}{2}\rho_\infty V_\infty^2 S C_L \quad \text{or} \quad V_\infty = \sqrt{\frac{2W}{\rho_\infty S C_L}}$$

- Putting these together gives:

$$P_R = T_R V_\infty = \sqrt{\frac{2W^3 C_D^2}{\rho_\infty S C_L^3}} \propto \frac{1}{C_L^{3/2}/C_D}$$

- Thus, for minimum power required, we want to have the maximum of $C_L^{3/2}/C_D$

Power Required (continued)

- Also, as before, we can split the thrust into profile and induced contributions:

$$P_R = T_R V_\infty = D V_\infty = q_\infty S\left(C_{D,0} + \frac{C_L^2}{\pi e AR}\right)V_\infty$$

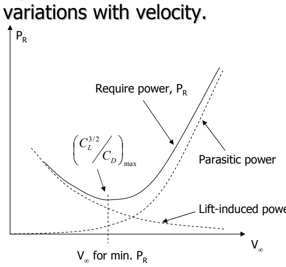
- Or, upon expanding

$$P_R = \tfrac{1}{2}\rho_\infty V_\infty^3 S C_{D,0} + \frac{2KW^2}{\rho_\infty V_\infty S}$$

- The first term is the zero-lift or parasitic power - I.e. the power required to overcome friction.
- The second term is the lift-induced power - I.e. the power required to produce lift.

Power Required (continued)

- When plotted versus velocity, we get a graph qualitatively similar to the thrust curves, but with different variations with velocity.



Power Required (continued)

- To find the minimum power required point, find the point where the slope is zero:

$$\left.\frac{dP_R}{dV_\infty}\right|_{P_{R,\min}} = \tfrac{3}{2}\rho_\infty V_\infty^2 S\left(C_{D,0} - \frac{\tfrac{1}{3}C_L^2}{\pi e AR}\right) = 0$$

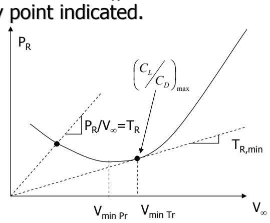
- Which turns out to be when:

$$\boxed{C_{D,0} = \frac{\tfrac{1}{3}C_L^2}{\pi e AR} = \tfrac{1}{3}C_{D,i}}$$ For minimum power required

- Thus, the point of minimum power required is when the induced drag is three times the parasitic drag.
- This is slower than the minimum thrust point which occurred when they were equal!

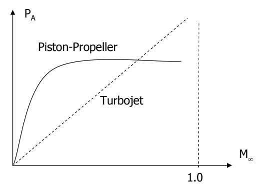
Power Required (continued)

- The slope of any line from the origin on this plot has a slope equal to the required thrust.
- Thus, the minimum T_R (minimum slope) occurs at the tangency point indicated.



Power Available

- Power available can be obtained from multiplying the thrust available by the velocity.
- As with thrust available, how power available varies depends upon the powerplant type:
- For turbojets, power increases almost linearly.
- For piston-propellers, power increases rapidly at low speeds, but is nearly constant for much of the flight regime.



Power Available (continued)

- As mentioned, a common measurement of piston and turboprop engine power output is horsepower.

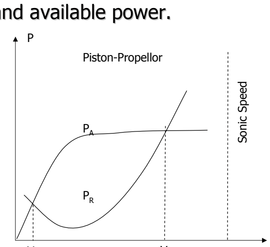
$$1\text{ hp} = 550\text{ ft}\cdot\text{lb/sec} = 746\text{ W}$$

- Also, propeller aircraft all the power produced by the engine does not go into producing thrust - some is lost by the inefficiencies of the propeller.
- To account for this, we introduce two new terms:
  - the propeller efficiency, η. ($\eta \le 1.0$)
  - the engine power, P, called the shaft brake horsepower, bhp (or shp).

$$\text{hp}_\text{A} = \eta \cdot \text{bhp} \qquad \text{or} \qquad P_A = \eta P$$

Power Available (continued)

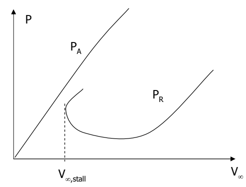
- As with thrust, the minimum and maximum flight velocities can be found from the intersection of required and available power.



Power Available (continued)

- One final note: all of the figures shown so far always have an intersection of P_A and P_R at low speeds.
- In fact, under many conditions, this intersection doesn't exist due to aircraft stall:
- In this situation, the lowest possible velocity for level flight is dictated by the C_{Lmax}:

$$V_{\infty,stall} = \sqrt{\frac{2W}{\rho_\infty S C_{L\max}}}$$



Altitude Effects on Power

- Earlier, it was shown how velocity and power required depended upon ρ, C_L, C_D, S and W

$$V_\infty = \sqrt{\frac{2W}{\rho_\infty S C_L}} \qquad P_R = \sqrt{\frac{2W^3 C_D^2}{\rho_\infty S C_L^3}}$$

- If we consider other altitudes, only ρ will change, the other values being independent of altitude.
- Thus, defining reference quantities at sea level conditions:

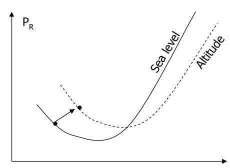
$$V_0 = \sqrt{\frac{2W}{\rho_0 S C_L}} \qquad P_{R,0} = \sqrt{\frac{2W^3 C_D^2}{\rho_0 S C_L^3}}$$

Altitude Effects (continued)

- And to relate altitude conditions to sea level:

$$V_{alt} = V_0\sqrt{\frac{\rho_0}{\rho}} = \frac{V_0}{\sqrt{\sigma}} \qquad P_{R,alt} = P_{R,0}\sqrt{\frac{\rho_0}{\rho}} = \frac{P_{R,0}}{\sqrt{\sigma}}$$

- These relations indicate a shift in the power required curve to higher powers and higher velocities as altitude increases (or as σ decreases)



Altitude Effects (continued)

- Available power also varies with altitude.
- At constant velocity, it is reasonable to assume that P_A and T_A vary linearly with density since they both will increase with mass flow rate:

$$P_{A,alt} = P_{A,0}\left(\frac{\rho_\infty}{\rho_0}\right) = \sigma P_{A,0}$$

- (Note: some references argue that turbojet and turbofan performance varies with pressure ratio, p_∞/p_o!)
- For piston engines, supercharging, the pre-compression of intake air, can eliminate the density variation of power up to some altitude.

Altitude Effects (continued)

- The combined effect of altitude on P_A and P_R is to reduce the maximum velocity, and increase the minimum velocity.