Radiation Basics - Light properties

- Remember that light is composed of photons which behave as both particles and waves.
- As waves, we can define the type of radiation by its wavelength, λ (μm), or frequency, ν (hz), which are related by:

$$\lambda\, v = c \equiv \text{speed of sound} = 3\text{x}10^{8}\ \text{m/sec (vacuum)}$$

- The energy of each photon is proportional to it frequency by:

$$E = hv \qquad h \equiv \text{Planck's constant} = 6.625\text{x}10^{-34}\ \text{J sec}$$

- Photons also have mass in proportion to their energy, but we will not have to consider this.

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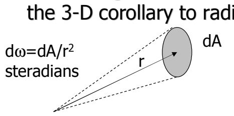
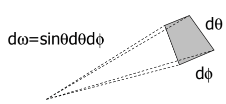
Radiation Basics - Intensity

- Spectral Intensity, I_λ: the radiant energy per unit area per unit solid angle per unit wavelength.
- The area of interest is the surface area perpendicular to the radiation direction.
- A solid angle is measured in steradians (sr) which is the 3-D corollary to radians.





- We can also further sub-classify the intensity as either that emitted, $I_{\lambda,e}$, or incident, $I_{\lambda,I}$, from the surroundings.

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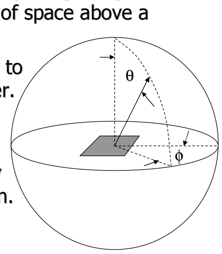
Radiation Basics - Fluxes

- Energy fluxes can be obtained by integrating the Intensity over the hemisphere of space above a surface.
- Integrate the emitted intensity to get the spectral emissive power.

$$E_\lambda = \int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,e} \cos\theta \sin\theta \, d\theta \, d\phi$$

- Integrate the incident intensity to get the spectral irradadiation.

$$G_\lambda = \int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,i} \cos\theta \sin\theta \, d\theta \, d\phi$$



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Radiation Basics - Fluxes (cont)

- Since intensity has the units of W/m²/st/μm, these two terms have units of W/m²/μm.
- If however, we also integrate over all possible frequencies, we can get the total emissive power and total irradiation:

$$E = \int_0^\infty E_\lambda d\lambda \qquad G = \int_0^\infty G_\lambda d\lambda$$

- These properties have units of W/m² and thus are heat fluxes!
- Also, note the terminology - spectral indicates a property which is a function of wavelength, total indicates properties which are not.

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Radiation Basics - Fluxes (cont)

- There is one other flux term of interest - that called Radiosity.
- The radiation which leaves a surface is not just emission, but also reflection of incident radiation.
- Thus, if $I_{\lambda,e+r}$ is the spectral intensity of both emission and reflection, then the spectral radiosity is:

$$J_\lambda = \int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,e+r} \cos\theta \sin\theta \, d\theta \, d\phi$$

- And the total radiosity follows as:

$$J = \int_0^\infty J_\lambda d\lambda$$

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Radiation Basics - Fluxes (cont)

- One final note: if the intensity is independent of direction it is called diffuse.
- In this case, the integration of the remaining angular terms is just:

$$\int_0^{2\pi} \int_0^{\pi/2} \cos\theta \sin\theta \, d\theta \, d\phi = \pi$$

- As a result, for diffuse radiation:

$$E_\lambda = \pi I_\lambda \qquad G_\lambda = \pi I_{\lambda,i} \qquad J_\lambda = \pi I_{\lambda,e+r}$$

- Later we will discuss when this simplification is justified.

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Blackbodies

- An idealized body used in radiation analysis is that of a so called Blackbody.
- A blackbody has the following properties:
  - They absorb all incident radiation regardless of wavelength and direction.
  - They emit radiation diffusely and over a continuous range of wavelengths at predictable spectral intensities defined later.
  - The radiation emitted from a blackbody at any frequency is the maximum theoretically possible for a given temperature.
- You should realize that these are called black bodies because of the absorption property.
- However, at high temperatures a blackbody will glow cherry or even white hot!

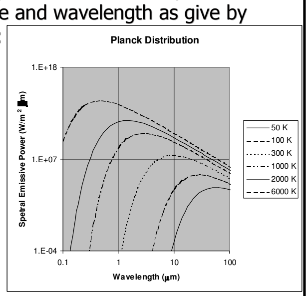
Blackbodies (cont)

- The spectral emission power of a blackbody is a function of temperature and wavelength as give by the Planck distribution:

$$E_{\lambda,b} = \frac{2\pi\, hc^2}{\lambda^5\left[\exp(hc/\lambda\, kT)-1\right]}$$

$k =$ Boltzmann constant

$= 1.38x10^{-23}\ J/K$



Blackbodies (cont)

- If only the total emissive power is desired, the Planck distribution can be integrated over wavelength to get:

$$E_b = \int_0^\infty \frac{2\pi\, hc^2}{\lambda^5\left[\exp(hc/\lambda\, kT)-1\right]} d\lambda \;= \sigma\, T^4$$

- Which is called the Stefan-Boltzmann Law and σ is the Stefan-Boltzmann constant, $5.670x10^{-8}$ W/m^2 K^4.

Band Emissions

- On occasion, it is advantageous to consider the emission or absorption over give a range of wavelengths.
- For example, if we were interested in the visible spectrum, we would be interested in wavelengths between 0.4 and 0.7 μm.
- To simplify this process, we define the blackbody functions as:

$$E_{b(\lambda_1 \to \lambda_2)} = \int_{\lambda_1}^{\lambda_2} E_{\lambda,b} d\lambda = \int_0^{\lambda_2} E_{\lambda,b} d\lambda - \int_0^{\lambda_1} E_{\lambda,b} d\lambda = E_{b(0\to\lambda_2)} - E_{b(0\to\lambda_1)}$$

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Band Emissions (cont)

- Note that the blackbody function is nothing more than the fraction of the total emissive power between the two indicated wavelengths.
- In Table 8.1, the book has the blackbody functions tabulated versus the product of wavelength and T.
- Thus to find the total emission power in the visible spectrum (0.4-0.7μm) from a black body at 5000^{o}K:

$\lambda_1 T$ = (0.4 μm)(5000^{o}K) = 2000 μm ^{o}K

$\lambda_2 T$ = (0.7 μm)(5000^{o}K) = 3500 μm ^{o}K

From Table 12.1: $E_{b(0\to\lambda 1)}/\sigma T^4=0.06672$ $E_{b(0\to\lambda 2)}/\sigma T^4=0.3829$

$E_{visible}=E_{b(\lambda 1 \to\lambda 2)}= (E_{b(0\to\lambda 2)}-E_{b(0\to\lambda 1)})=$

$(5.670\text{X}10^{-8}$ W/m^2 K^4)(5000^{o}K)^4(.3829-0.06672)=11.2 MW/m^2

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Emissivity

- Real surfaces differ from blackbodies in three ways:
  - Their total radiation at any wavelength is always less that that of a blackbody for the same frequency.
  - The surface emission tends to be wavelength selective rather than continuous.
  - The surface emission is not usually diffuse.
- To account for these variations, we introduce the spectral, directional emissivity as defined by:

$$\varepsilon_{\lambda,\theta}(\lambda,\theta,\phi,T) = \frac{I_{\lambda,e}(\lambda,\theta,\phi,T)}{I_{\lambda,b}(\lambda,T)}$$

- Where $0 \le \varepsilon_{\lambda,\theta}(\lambda,\theta,\phi,T) \le 1$

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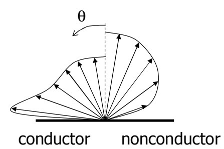
Emissivity (cont)

- By integrating over all frequencies, we can obtain the total, directional emissivity - a measure of the non-diffuseness of the radiation:

$$\varepsilon_\theta(\theta,\phi,T)=\frac{I_e(\theta,\phi,T)}{I_b(T)}$$

- Generally, this property is not a function of ϕ, but only of θ.
- How the emission varies with angle from the surface also depends upon the electrical conductance properties of the material as shown:

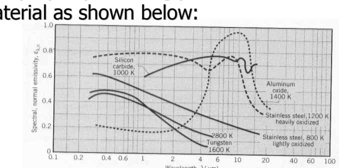


Emissivity (cont)

- Or by integrating overall all directions, we can obtain the spectral, hemispherical emissivity - a measure of the total emission at a given wavelength:

$$\varepsilon_\lambda(\lambda,T)=\frac{E_\lambda(\lambda,T)}{E_{\lambda,b}(\lambda,T)}$$

- This property is strongly a function of the type of material as shown below:



Emissivity (cont)

- However, for engineering purposes, it is usually sufficient to consider the total, hemispherical emissivity after integrating over all wavelengths and directions:

$$\varepsilon(T)=\int_0^\infty \varepsilon_\lambda(\lambda,T)d\lambda=\frac{E(T)}{E_b(T)}$$

- This value is usually with 30% of the emissivity in the direction normal to the surface for conductors and 5% for non-conductors.
- In Appendix Table A-10 is tabulated either the total normal emissivity for a wide range of materials.

Emissivity (cont)

- Final notes:
  - The emissivity of non-conductors is generally much higher than that for conductors.
  - For metals, the emissivity is strongly dependant upon the surface condition. Polished surfaces have much lower emissivities than rough or oxidized ones.
  - The emissivity for conductors generally increases with temperature.
  - For nonconductors, the emissivity varies differently with temperature for different materials - no general trend exists.

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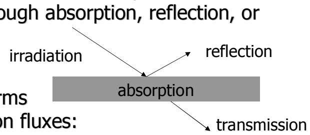
Irradiation

- The flux of energy to a surface may interact with it in one of 3 ways - through absorption, reflection, or transmission.



- If follows that in terms of spectral irradiation fluxes:

$$G_\lambda = G_{\lambda,ref} + G_{\lambda,abs} + G_{\lambda,tr}$$

- From this we can define 3 new properties:

Spectral, hemispherical <u>absorptivity</u> $\alpha_\lambda = \dfrac{G_{\lambda,abs}}{G_\lambda}$

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Irradiation (cont)

Spectral, hemispherical <u>reflectivity</u> $\rho_\lambda = \dfrac{G_{\lambda,ref}}{G_\lambda}$

Spectral, hemispherical <u>transmissivity</u> $\tau_\lambda = \dfrac{G_{\lambda,tr}}{G_\lambda}$

- Where it must be true that:

$$1 = \rho_\lambda + \alpha_\lambda + \tau_\tau$$

- Similarly, we could integrate over all frequencies to define the total, hemispherical quantities:

$$\alpha = \frac{G_{abs}}{G} \quad \rho = \frac{G_{ref}}{G} \quad \tau = \frac{G_{tr}}{G} \qquad 1 = \rho + \alpha + \tau$$

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Irradiation (cont)

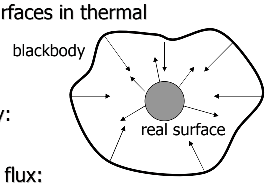
- Note that for opaque materials, $\tau = \tau_\lambda = 0$. Thus:

$$\alpha_\lambda = 1 - \rho_\lambda \qquad \alpha = 1 - \rho$$

- Also, note the large dependence upon frequency for non-conducting materials.
- Conductors, due to the many possible energy states of the free electrons, are much less frequency dependant.

Kirchoff's Law

- Consider a real surface totally enclosed by a black body and with the two surfaces in thermal equilibrium.
- The irradiation of the surface would be the emission of the blackbody:



$$G = E_b(T_s)$$

- And the absorbed energy flux:

$$q''_{abs} = \alpha\, E_b(T_s)$$

- The emission flux from the surface would be:

$$q''_e = \varepsilon\, E_b(T_s)$$

Kirchoff's Law (cont)

- However, for equilibrium, the net flux coming and going must be zero. Thus:

$$q''_{abs} = q''_e \qquad \alpha\, E_b(T_s) = \varepsilon\, E_b(T_s)$$

- Or, more simply:

$$\boxed{\alpha = \varepsilon}$$ Kirchoff's Law

- To be precise, this relation is only valid for radiation exchange with a blackbody. For the more general case of exchange between two real surfaces:

$$\alpha_{\lambda,\theta} = \varepsilon_{\lambda,\theta}$$

- Where both the wavelength and direction of the irradiation must be accounted for.

Gray Surfaces

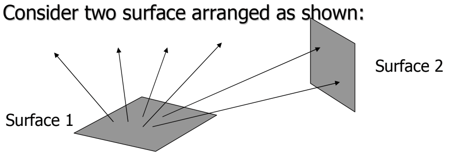
- For engineering applications, it is usually sufficient to consider only Kirchoff's Law for the total, hemispherical properties, I.e.:

$$\alpha = \varepsilon$$

- This assumption is sometimes called the Gray Surface approximation.
- It is valid if two conditions are true:
  - The irradiation and emission are both diffuse.
  - The spectral absorptivity and emissivity are nearly constant across the bulk of the irradiation and blackbody emissive wavelength spectrum.
- All of our future work will use this approximation!

View Factors

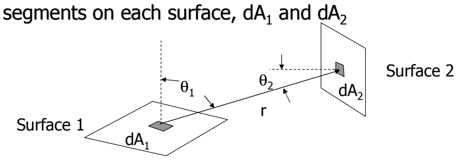
- After resolving the surface radiative surface properties, the next task is to tackle the surface geometry issues.
- Consider two surface arranged as shown:



- We define the fraction of radiant energy leaving surface 1 (either by emission or reflection) which is actually incident on 2 as the View Factor, F_{12} .

View Factors (cont)

- To calculate the view factor, consider smaller area segments on each surface, dA_1 and dA_2



- The radiation intensity leaving dA_1 in the direction of dA_2 is $I_1 dA_1 \cos\theta_1$ (Watts/steradian)
- Thus, the flux striking dA_2 is: $I_1 dA_1 \cos\theta_1 \left(\frac{dA_2 \cos\theta_2}{r^2} \right)$

View Factors (cont)

- To get the total heat transfer, integrate over both surfaces:

$$q_{1\to 2} = \int_{A_1}\int_{A_2} I_1 \left(\frac{\cos\theta_1 \cos\theta_2}{r^2} \right) dA_2 dA_1$$

- Now, if the radiation is diffuse and uniform over surface 1, the intensity is related to the radiosity by $I_1 = J_1/\pi$.
- But the total flux leaving surface 1 is $A_1 J_1$. Thus the view factor is:

$$F_{12} = \frac{q_{1\to 2}}{A_1 J_1} = \frac{1}{A_1} \int_{A_1}\int_{A_2} \left(\frac{\cos\theta_1 \cos\theta_2}{\pi\, r^2} \right) dA_2 dA_1$$

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View Factors (cont)

- Or, for the view factor for any surface, i, to any other surface, j, :

$$F_{ij} = \frac{1}{A_i} \int_{A_i}\int_{A_j} \left(\frac{\cos\theta_i \cos\theta_j}{\pi\, r^2} \right) dA_j dA_i$$

- In practice, this integral can be a real pain to calculate.
- Thus, the book provides tabulated and graphical results for a variety of simple geometries in Table 8.2 and Figures 8-12 to 8-16.

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View Factor Algebra

- The selection of geometry cases provided in the Textbook seems limited.
- However, some simple algebraic rules allows us to build more complex cases out of simple ones.
- The simplest rule is that of reciprocity. If F_{ij} is as give before and F_{ji} is:

$$F_{ji} = \frac{1}{A_j} \int_{A_j}\int_{A_i} \left(\frac{\cos\theta_j \cos\theta_i}{\pi\, r^2} \right) dA_i dA_j$$

- Then it must be true that:

$$A_i F_{ij} = A_j F_{ji}$$

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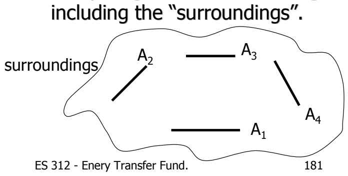
View Factor Algebra (cont)

- Next, the sum of all shape factors for any surface must be equal to 1.0:

$$1 = F_{i1} + F_{i2} + F_{i3} + \cdots = \sum_{k=1}^{n} F_{ik}$$

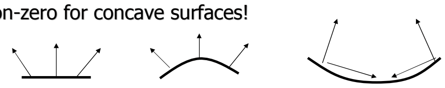
- Note that this summation should extend over everything a surface exchanges energy with, including the "surroundings".



$$1 = F_{11} + F_{12} + F_{13} + F_{14} + F_{1surroundings}$$

View Factor Algebra (cont)

- Also note that there is a "self view factor", F_{ii}. For planar or convex surfaces it is zero. But it will be non-zero for concave surfaces!

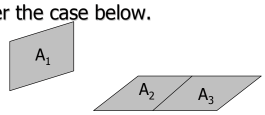


$F_{ii} = 0$ $F_{ii} = 0$ $F_{ii} \neq 0$

- Note that the two rules provided so far just reduce the number of different view factors that must be calculated through integration (or charts).
- The last rule allows us to tackle difficult geometries.

View Factor Algebra (cont)

- The last rule is that view factors should be additive when dealing with composite surfaces.
- Consider the case below.



- The view factor from surface 1 to the composite surface 2,3, should be the same as the sum of the view factor from 1 to 2 and from 1 to 3:

$$F_{1(2,3)} = F_{12} + F_{13}$$

View Factor Algebra (cont)

- To generalize for any composite surface j made up of sub surfaces k=1→n: $F_{i(j)} = \sum_{k=1}^{n} F_{ik}$
- Note, however, that the reverse view factor, by the reciprocity rule is: $A_{(j)} F_{(j)i} = \sum_{k=1}^{n} A_k F_{ki}$
- Or
$$F_{(j)i} = \frac{\sum_{k=1}^{n} A_k F_{ki}}{A_{(j)}} = \frac{\sum_{k=1}^{n} A_k F_{ki}}{\sum_{k=1}^{n} A_k}$$
- The best way to see how to apply these rules is by examples!

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Radiant Exchange - Black Bodies

- We have looked at surface properties, then at geometric arrangements, now lets get around to actually calculating radiant fluxes.
- Begin be considering just the case where all the surfaces of interest are black.
- This is a simplification because we do not need to deal with reflected radiation! All the radiation which reaches a surfaces will be absorbed.
- From the view factor calculations, we found that the heat rate from one surface, i, to another, j, was:
$$q_{i \to j} = F_{ij} A_i J_i$$

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Radiant Exchange - Black Bodies (cont)

- But, if surface i is black, its radiosity is just the black body emissive power:
$$q_{i \to j} = F_{ij} A_i J_i = F_{ij} A_i E_{bi} = F_{ij} A_i \sigma T_i^4$$
- Similarly, the heat transfer rate of from surface j to surface i is:
$$q_{j \to i} = F_{ji} A_j J_j = F_{ji} A_j E_{bj} = F_{ji} A_j \sigma T_j^4$$
- So, the net rate exhanged by i and j is (i loosing energy being positive):
$$q_{ij} = q_{i \to j} - q_{j \to i} = F_{ij} A_i \sigma T_i^4 - F_{ji} A_j \sigma T_j^4$$
- Or, using reciprocity: $q_{ij} = F_{ij} A_i \sigma \left(T_i^4 - T_j^4 \right)$

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Radiant Exchange - Black Bodies (cont)

- If we considered all exchanges between surface i and any other surface, we could write:

$$q_i = \sum_{j=1}^{n} q_{ij} = \sum_{j=1}^{n} F_{ij} A_i \sigma \left(T_i^4 - T_j^4\right)$$

- Some final notes:
  - The surface heat fluxes are negatives of each other, I.e.

  $$q_{ij} = -q_{ji}$$

  - It is pretty easy to make one of the surfaces the "surroundings" which is normally consider to be black.
  - Realize that we have neglected the medium between the surfaces (like air or a vacuum). These mediums are considered to be "nonparticiating".

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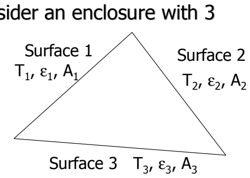
Radiant Exchange - Gray Surfaces

- Now lets consider radiant exchange between gray surfaces.
- The main differences from blackbodies is that we must consider the emissivity of each surface as well as the reflected, non-absorbed radiation flux.
- To simplify geometry, consider an enclosure with 3 sides as shown:
- Each surface has its own, uniform temperature and surface properties.
- View factors can be found from Table 13.1



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Radiant Exchange - Gray Surfaces (cont)

- At each surface, the net rate away from the surface is the difference between the radiosity and irradiation:

$$q_i = A_i \left(J_i - G_i\right)$$

- But the radiosity itself is the sum of the emission power and the reflected irradiation:

$$J_i = E_i + \rho_i G_i = \varepsilon_i E_{bi} + \rho_i G_i$$

- Solving for G_i and substituting into the rate equation with the added assumption that $\rho = 1-\alpha = 1-\varepsilon$:

$$G_i = \frac{J_i - \varepsilon_i E_{bi}}{1-\varepsilon_i} \qquad \Rightarrow \qquad q_i = A_i \left(J_i - \frac{J_i - \varepsilon_i E_{bi}}{1-\varepsilon_i} \right)$$

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Radiant Exchange - Gray Surfaces (cont)

- Or, after rewriting:

$$q_i = \frac{\varepsilon_i A_i}{1-\varepsilon_i}\left(E_{bi} - J_i\right)$$

- However, this is just the rate away from one surface.
- The exchange rate between two surfaces is the same as before:

$$q_{ij} = q_{i\to j} - q_{j\to i} = F_{ij} A_i J_i - F_{ji} A_j J_j$$

- Or, by reciprocity:

$$q_{ij} = F_{ij} A_i \left(J_i - J_j\right)$$

Radiant Exchange - Gray Surfaces (cont)

- If we think of the E_{bi}'s and J_i's as the potential for radiant heat transfer, then we can define the resistances to heat transfer as:

$$q_i = \frac{\left(E_{bi} - J_i\right)}{R_i} \qquad R_i = \frac{1-\varepsilon_i}{\varepsilon_i A_i} \quad \text{Surface resistance}$$

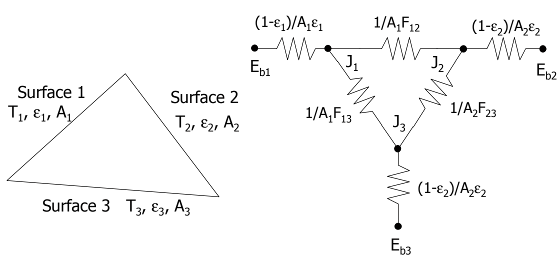
$$q_{ij} = \frac{\left(J_i - J_j\right)}{R_{ij}} \qquad R_{ij} = \frac{1}{F_{ij} A_i} \quad \text{Shape resistance}$$

- Also, not that by reciprocity:

$$R_{ij} = \frac{1}{F_{ij} A_i} = \frac{1}{F_{ji} A_j} = R_{ji}$$

Radiant Exchange - Gray Surfaces (cont)

- These results allows us to make the following electrical analogy:



Radiant Exchange - Gray Surfaces (cont)

- We solve this type of circuit by requiring the net flow rate into any of the nodes, Ji, to be zero:

$$0 = q_i - \sum_{j}^{j \neq i} q_{ij} = \frac{\varepsilon_i A_i}{1-\varepsilon_i}\left(E_{bi} - J_j\right) - \sum_{j}^{j \neq i} F_{ij} A_i \left(J_i - J_j\right)$$

- Or, for the 3 nodes we have in this case:

$$J_1: \quad 0 = \frac{1}{R_1}\left(E_{b1} - J_1\right) - \frac{1}{R_{12}}\left(J_1 - J_2\right) - \frac{1}{R_{13}}\left(J_1 - J_3\right)$$

$$J_2: \quad 0 = \frac{1}{R_2}\left(E_{b2} - J_2\right) - \frac{1}{R_{21}}\left(J_2 - J_1\right) - \frac{1}{R_{23}}\left(J_2 - J_3\right)$$

$$J_3: \quad 0 = \frac{1}{R_3}\left(E_{b3} - J_3\right) - \frac{1}{R_{31}}\left(J_3 - J_1\right) - \frac{1}{R_{32}}\left(J_3 - J_2\right)$$

Radiant Exchange - Gray Surfaces (cont)

- The problem of interest is when surface temperatures are known the surface heating rates are desired.
- In this case, the E_{bi}'s are also known since: $E_{bi} = \sigma T_i^4$.
- Thus, we have 3 unknowns, J_1, J_2, and J_3, which we find by solving the set of linear equations.
- After finding the J_i's, the heat rate to any one of the surface is just:

$$q_i = \frac{\left(E_{bi} - J_i\right)}{R_i}$$

- Can you see how to extend this method to more than 3 surfaces?