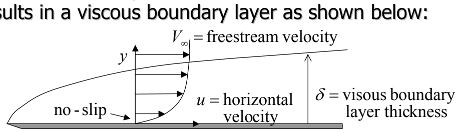
## Convection

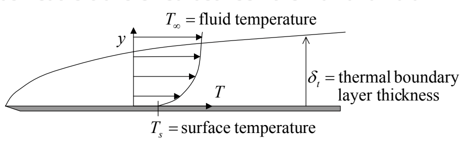
- Finally, lets look at the remaining form of heat transfer - Convection.
- Remember from when we began heat transfer that convection is a combination of two mechanisms:
  - Conduction of energy between fluid and solid at the wall.
  - Advection of energy carried by the moving fluid.
- The viscous no-slip condition at the wall interface results in a viscous boundary layer as shown below:






## Convection (cont)

- At the same time, a thermal boundary layer develops as heat is transferred between the wall and fluid:




- Note that at the surface, T(x,y,z) = $T_s$ - the thermal equivalent to no-slip.
- This thermal equilibrium is due to both the need for continuity and the low (zero!) flow velocities near the wall allowing time to equilibrate.



## Convection (cont)

- Also, because of the no-slip condition, the process right at the fluid-wall interface is just conduction.
- Thus, to find the heat flux between fluid and wall at any point along the surface:

$$q'' = -k_f \left( \frac{\partial T}{\partial y} \right)_{\text{wall}} \qquad k_f \equiv \text{fluid conductivity}$$

- But, by Newton's Law of Cooling: $q'' = h(T_s - T_\infty)$
- The point of these chapters is to learn how to determine h from the fluid flow problem.
- From the two equations above, this means finding the solution to:

$$\boxed{h = \frac{-k_f (\partial T / \partial y)_{wall}}{(T_s - T_\infty)}}$$

## Convection (cont)

- What we need then is to find how the temperature varies normal to the wall!
- To do this, we need to solve the fluid equations of motion - or make use of experimental results.
- Realize that the convective coefficient, h, is a function of surface location and may vary with x & y.
- Also, we will usually want the average coefficient over an entire surface:

$$\overline{h} = \frac{1}{A_s} \int_{A_s} h(x, y)\, dA_s$$

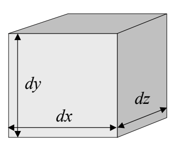
- This average convection coefficient is the one we have already used in our previous calculations!



## Conservation Equations

- Since we now know that we need to solve the fluid problem, let's look a the governing equations.
- We will need to conserve mass, momentum (3-directions), and energy in the boundary layer (B.L.).
- If we consider a small control volume in the B.L., a general statement of conservation for steady state is:

$$\begin{bmatrix} \text{The net rate at which} \\ \text{a property flow out} \end{bmatrix} = \begin{bmatrix} \text{The rate of production} \\ \text{of that property inside} \end{bmatrix}$$






## Conservation Equations (cont)

- Some textbooks go through a length development of conservations equations for the most general case: 3-D, compressible, fully viscous.
- The resulting very complex, equations are usually known as the Navier-Stokes Equations.
- In class, we will do a simpler development by making assumptions valid for most boundary layers.
- I.e.
  - 2-D flow — Viscous normal to the surface
  - Incompressible — Neglecting higher order terms.
- The resulting equations are called the Boundary Layer Equations and only apply to the viscous layer!

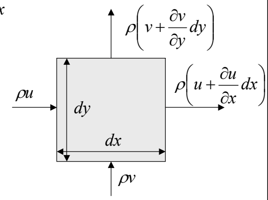
## Mass Conservation

- Apply this conservation rule to mass for the 2-D case below.
- The net rate of mass outflow is:

$$\rho\left(u+\frac{\partial u}{\partial x}dx\right)dy+\rho\left(v+\frac{\partial v}{\partial y}dy\right)dx$$
$$-\rho\, u dy-\rho\, v dy$$

- Since there is no mass production, we get:

$$\boxed{\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0}$$

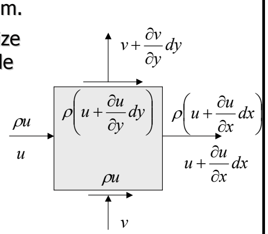





## Momentum Conservation

- Now consider momentum conservation - in particular conservation of X momentum.
- In the diagram shown, realize that ρu is the property, while both u and v are flux rates!
- Thus, the next flux out is:

$$\rho\left(u+\frac{\partial u}{\partial x}dx\right)^2 dy+$$
$$\rho\left(u+\frac{\partial u}{\partial y}dy\right)\left(v+\frac{\partial v}{\partial y}dy\right)dx$$
$$-\rho\, u^2 dy-\rho\, uvdx$$






## Momentum Conservation (cont)

- After expanding, canceling terms, and dropping products of differentials (like $\frac{\partial u}{\partial y}\frac{\partial v}{\partial y}$) as negligible, we get:

$$\text{X momentum flux}=\left(2\rho\, u\frac{\partial u}{\partial x}+\rho\, v\frac{\partial u}{\partial y}+\rho\, u\frac{\partial v}{\partial y}\right)dxdy$$

- Or, since by mass conservation: $\frac{\partial u}{\partial x}=-\frac{\partial v}{\partial y}$

$$\text{X momentum flux}=\left(\rho\, u\frac{\partial u}{\partial x}+\rho\, v\frac{\partial u}{\partial y}\right)dxdy$$

- Doing the same process for Y momentum gives:

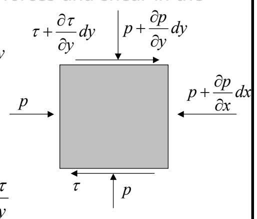
$$\text{Y momentum flux}=\left(\rho\, v\frac{\partial v}{\partial y}+\rho\, u\frac{\partial v}{\partial x}\right)dxdy$$

## Momentum Conservation (cont)

- Momentum is produced by forces acting on the C.V.
- If we consider only pressure forces and shear in the x direction, then:

$$\text{X production} = \left(-\frac{\partial p}{\partial x} + \frac{\partial \tau}{\partial y}\right) dxdy$$

$$\text{Y production} = -\frac{\partial p}{\partial y} dxdy$$




- Setting net flux equal to production gives:

$$\text{X}: \quad \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial \tau}{\partial y}$$

$$\text{Y}: \quad \rho v \frac{\partial v}{\partial y} + \rho u \frac{\partial v}{\partial x} = -\frac{\partial p}{\partial y}$$



## Momentum Conservation (cont)

- However, Newton's Law of Viscosity states that:

$$\tau = \mu \frac{\partial u}{\partial y} \qquad \mu = \text{fluid viscosity}$$

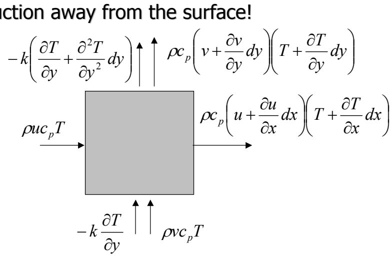
- Also, experimental evidence shows that in most B.L.'s, both dp/dy and v are small. As a result, the Y momentum eqn. is much less important than the X.
- Thus, it is sufficient in a boundary layer to express momentum conservation by:

$$\boxed{\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2}}$$



## Energy Conservation

- Lastly, consider the fluxes of energy out of our C.V.
- Note that there will not only be advection, but also conduction away from the surface!



## Energy Conservation (cont)

- Summing the fluxes gives:

$$\rho c_p\left(u+\frac{\partial u}{\partial x}dx\right)\left(T+\frac{\partial T}{\partial x}dx\right)dy+\rho c_p\left(v+\frac{\partial v}{\partial y}dy\right)\left(T+\frac{\partial T}{\partial y}dy\right)dx$$
$$-k\left(\frac{\partial T}{\partial y}+\frac{\partial^2 T}{\partial y^2}dy\right)dx-\rho u c_p T dy-\rho v c_p T dx+-k\frac{\partial T}{\partial y}dx$$

- Or after canceling and dropping higher order terms:

$$\left[\rho c_p u\frac{\partial T}{\partial x}+\rho c_p v\frac{\partial T}{\partial y}+\rho c_p T\underbrace{\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)}_{0}-k\frac{\partial^2 T}{\partial y^2}\right]dxdy$$

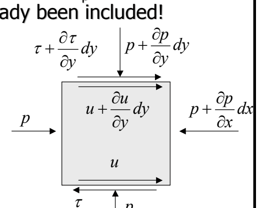
- But, by mass conservation, the 3rd term is zero!



## Energy Conservation (cont)

- Energy is generated by the rate at which work is done by surface forces.
- We have both pressure and friction forces, but since we used enthalpy for the energy, $h=c_pT$, pressure work (or flow work) has already been included!
- Without going into details, the total work rate due to friction is just:

$$\mu\left(\frac{\partial u}{\partial y}\right)^2 dxdy$$






## Energy Conservation (cont)

- Thus, our energy conservation is just:

$$\rho c_p u\frac{\partial T}{\partial x}+\rho c_p v\frac{\partial T}{\partial y}-k\frac{\partial^2 T}{\partial y^2}=\mu\left(\frac{\partial u}{\partial x}\right)^2$$

- And to summarize all of our conservation equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$
$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}$$
$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial x}\right)^2$$

2-D, Incompressible, Boundary Layer Equations

## Non-Dimensional Equations

- Rather than attempt to solve the governing Boundary Layer equations just derived, instead, consider non-dimensionalizing them.
- To obtain non-dimension distances and velocities, divide by some characteristic length and the freestream velocity, respectively:

$$\bar{x} = \frac{x}{L} \qquad \bar{y} = \frac{y}{L} \qquad \bar{u} = \frac{u}{V_\infty} \qquad \bar{v} = \frac{v}{V_\infty}$$

- For pressure and temperature, use slightly more complex definitions:

$$\bar{p} = \frac{p}{\gamma p_\infty} \qquad \bar{T} = \frac{T - T_s}{T_\infty - T_s} = \frac{T - T_s}{\Delta T}$$



## Non Dimensional Equations (cont)

- While the other properties are constants and are left alone (including density!).
- After inserting these definitions into our B.L. eqns:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = -\left(\frac{\gamma p_\infty}{\rho_\infty V_\infty^2}\right)\frac{\partial \bar{p}}{\partial \bar{x}} + \left(\frac{\mu}{\rho_\infty V_\infty L}\right)\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}$$

$$\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} = \left(\frac{k}{\rho_\infty c_p V_\infty L}\right)\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \left(\frac{\mu V_\infty}{\rho_\infty c_p L \Delta T}\right)\left(\frac{\partial \bar{u}}{\partial \bar{x}}\right)^2$$

- Now consider the grouping of terms inside the brackets.



## Non Dimensional Equations (cont)

- From gas dynamics, the first term resolves into a function of the Mach number, M:

$$\frac{\gamma p_\infty}{\rho_\infty V_\infty^2} = \frac{a_\infty^2}{V_\infty^2} = \frac{1}{M^2} \qquad \boxed{M = \frac{V_\infty}{a_\infty}}$$

- Literally, the Mach number is the ratio of the flow velocity to the speed of sound, a.
- Practically, the Mach number is a good measure of the compressibility of a fluid.
- For M < 0.3, the fluids density is constant for all practical purposes.
- For M > 0.3, changes in density must be taken into account.

## Non Dimensional Equations (cont)

- The second grouping of terms is called the <u>Reynolds number</u>, Re, based on the length L:

$$\frac{\mu}{\rho_\infty V_\infty L} = \frac{\upsilon}{V_\infty L} = \frac{1}{\mathrm{Re}_L} \qquad \boxed{\mathrm{Re}_L = \frac{\rho_\infty V_\infty L}{\mu}}$$

- The Reynolds number is the ratio of flow momentum to fluid viscosity.
- For low Re's (<100), viscosity dominates the flow and the viscous boundary layer is very thick.
- At high Re's (> 1e6), the B.L. becomes thin and viscous effects are only important near the surface.
- Note also the term $\upsilon=\mu/\rho$ which is called either the kinematic viscosity or the <u>viscous diffusivity</u>.



## Non Dimensional Equations (cont)

- The next group of terms can be written as the product of Reynolds number and a new term, the <u>Prandlt number</u>, Pr.

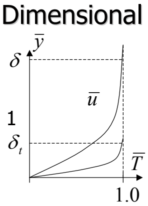
$$\frac{k}{\rho_\infty c_p V_\infty L} = \frac{\alpha}{V_\infty L} = \frac{\upsilon}{V_\infty L}\left(\frac{\alpha}{\upsilon}\right) = \frac{1}{\mathrm{Re}_L}\left(\frac{1}{\mathrm{Pr}}\right) \qquad \boxed{\mathrm{Pr} = \frac{\upsilon}{\alpha}}$$

- The Prandlt number is the ratio of viscous to thermal diffusivities.
- When large (as in oils), the viscous boundary layer is much larger than the thermal boundary layer.
- When small (as in liquid metals), the opposite is true.
- When Pr ~ 1 (as in gasses), they are about equal in size!

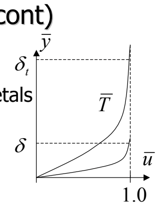


## Non Dimensional Equations (cont)

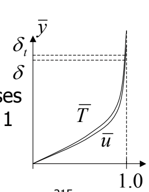
Oils
Pr >> 1




Liquid Metals
Pr << 1




Gasses
Pr ≅ 1



## Non Dimensional Equations (cont)

- Note that the product of $\text{Re}_L$ and Pr may be used by itself and is called the Peclet number, $\boxed{\text{Pe} = \text{Re}_L \Pr}$
- And, the last term is composed of the Reynolds number and the Eckert number, Ec:

$$\frac{\mu V_\infty}{\rho_\infty c_p L \Delta T} = \frac{\upsilon}{V_\infty L}\left(\frac{V_\infty^2}{c_p \Delta T}\right) = \frac{\text{Ec}}{\text{Re}_L} \qquad \boxed{\text{Ec} = \frac{V_\infty^2}{c_p\left(T_s - T_\infty\right)}}$$

- The Eckert number is the ratio of the flow energy to the B.L. enthalpy difference.
- Generally, the Eckert number is useful in cases where heating from flow friction (aeroheating) becomes important.



## Non Dimensional Equations (cont)

- With these definitions, our B.L. equations become:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{1}{M^2}\frac{\partial \bar{p}}{\partial \bar{x}} + \frac{1}{\text{Re}_L}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}$$

$$\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} = \frac{1}{\text{Re}_L \Pr}\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{\text{Ec}}{\text{Re}_L}\left(\frac{\partial \bar{u}}{\partial \bar{x}}\right)^2$$

- Thus, when solved, we would get solutions of u, v and T in the form:

$$\bar{u} = f\left(\text{M}, \text{Re}_L\right) \qquad \bar{v} = f\left(\text{M}, \text{Re}_L\right)$$

$$\bar{T} = f\left(\text{M}, \text{Re}_L, \Pr, \text{Ec}\right)$$



## Non Dimensional Equations (cont)

- However, the velocities and temperatures are not really what we are after!
- Instead, we would like to get viscous forces and heat transfer flux, $\tau$ and q".
- To get the surface shear force, use:

$$\tau = \mu \left.\frac{\partial u}{\partial y}\right|_{wall} = \frac{\mu V_\infty}{L}\left.\frac{\partial \bar{u}}{\partial \bar{y}}\right|_{wall} = \left(\rho V_\infty^2\right)\text{Re}_L \left.\frac{\partial \bar{u}}{\partial \bar{y}}\right|_{wall}$$

- This leads to a non-dimensional form of shear stress called the skin friction coefficient, $C_f$:

$$\boxed{C_f = \frac{\tau}{\left(\frac{1}{2}\rho V_\infty^2\right)}} = 2\,\text{Re}_L \left.\frac{\partial \bar{u}}{\partial \bar{y}}\right|_{wall}$$

## Non Dimensional Equations (cont)

- To get the heat flux, use:

$$q'' = -k_f \left.\frac{\partial T}{\partial y}\right|_{wall} = -\frac{k_f\left(T_\infty - T_s\right)}{L}\left.\frac{\partial \overline{T}}{\partial \overline{y}}\right|_{wall}$$

- This leads to a non-dimensional heat flux known as the Nusselt number, Nu:

$$\mathrm{Nu}_L = \frac{q'' L}{k_f\left(T_s - T_\infty\right)} = \left.\frac{\partial \overline{T}}{\partial \overline{y}}\right|_{wall}$$

- However, the heat transfer coefficient is given by:

$$q'' = h\left(T_s - T_\infty\right)$$

- Thus: $$\boxed{\mathrm{Nu}_L = \frac{hL}{k_f}}$$



## Non Dimensional Equations (cont)

- Note how the Nusselt number resembles the Biot number, Bi = hL/k, BUT DO NOT CONFUSE THEM!
- The Biot number is the ratio of external convection to internal conduction.
- The Nusselt number on the other hand is the ratio of the convective heat flux to that which would exist if the fluid conducted heat only (no fluid motion)!
- Thus, the Biot number uses k of the solid surface, while the Nusselt number uses k of the fluid.



## Non Dimensional Equations (cont)

- There is one final parameter to mention, the Stanton number, St: $\boxed{\mathrm{St} = \frac{h}{\rho c_p V_\infty}} = \frac{\mathrm{Nu}_L}{\mathrm{Re}_L\,\mathrm{Pr}}$
- The Stanton number is the ratio of energy flux normal to the wall to that tangent to it.
- Note that this is a corollary to the definition of the skin friction coefficient:

$$C_f = \frac{\tau}{\left(\frac{1}{2}\rho V_\infty^2\right)}$$

- Do to the similarity between viscous and thermal boundary layers, we might expect some relationship to exist between these two.

## Non Dimensional Equations (cont)

- In fact, such relationships due exist, usually in the form:

$$\mathrm{St} = \frac{\mathrm{Nu_L}}{\mathrm{Re_L\,Pr}} = f(C_f, \mathrm{Pr})$$

- This result is usually known as either the Reynolds or Chilton-Colburn analogy.
- The usefulness of this relation is that it is often easier to solve (or measure) the viscous boundary layer than the thermal one.
- With the above relation, once one solution is known, the other (viscous or thermal) is rapidly found.



## Non Dimensional Equations (cont)

- So, to summarize, our non-dimensional investigation indicates that, in the most general situation:

$$C_f = f(\mathrm{M}, \mathrm{Re}_L) \qquad \mathrm{Nu}_L = f(\mathrm{M}, \mathrm{Re}_L, \mathrm{Pr}, \mathrm{Ec})$$

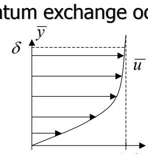
- Of course, any solution will also depend upon the geometric arrangement.
- Next we will begin to look at different flow situations and see the exact form the above functions take and how to apply them.
- Also, realize that the average values of these parameters over a surface would be more useful:

$$\overline{C}_f = \frac{1}{A_s}\int_{A_s} C_f\, dA_s \qquad \overline{\mathrm{Nu}}_L = \frac{1}{A_s}\int_{A_s} \mathrm{Nu}_L\, dA_s$$



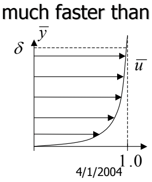
## Laminar/Turbulent Flow

- Viscous boundary layers come in two varieties: laminar and turbulent.
- At low Reynolds numbers, when viscosity is large compared to flow momentum, the internal fluid stresses act to hold the flow together.
- The result is a smooth, well defined boundary layer where heat and momentum exchange occurs on a molecular scale.
- This is laminar flow:



## Laminar/Turbulent Flow (cont)

- At high Reynolds numbers, flow momentum overcomes internal stresses and the flow begins to tumble - this is turbulence.
- The resulting flow is very unsteady and chaotic - but can be well defined by time averaged properties.
- Due to the tumbling, rolling motion of the fluid, heat and momentum are exchanged much faster than in laminar flow.
- As a result viscous and thermal boundary layers appear to be "fuller".






## Laminar/Turbulent Flow (cont)

- For internal flows (in pipes and ducts), the flow is either laminar or turbulent, based upon the $Re_D$.
- For external flows (over plates or tubes), the flow always begins laminar and, depending on body size and shape, becomes turbulent.
- Because of the different heat and momentum flux rates in the two types of flows, our correlations must account for which we are dealing with.
- Also, when using the correlations in the book, be sure to check the applicability of the equation - most of the correlations only apply to specific problems!



## Laminar Flow on Plates

- For laminar flow over a flat plate, it is possible to find an approximate solution to the 2-D Boundary Layer Equations if the plate is at a fixed wall temperature.
- The results depend upon a number of assumptions and simplifications, but agree very well with experimental evidence.
- The results show that at any point:

$$\delta = 5\sqrt{\frac{x\mu}{\rho V_\infty}} = \frac{5x}{\sqrt{\mathrm{Re}_x}} \qquad C_f(x) = \frac{0.644}{\sqrt{\mathrm{Re}_x}}$$

$$\delta_t = \frac{\delta}{\mathrm{Pr}^{1/3}} \qquad Nu_x = 0.332\,\mathrm{Re}_x^{\,1/2}\,\mathrm{Pr}^{1/3}$$

## Laminar Flow on Plates (cont)

- And, for a plate with dimension L in the flow direction, the average quantities are:

$$\overline{C}_f = \frac{1.328}{\sqrt{\mathrm{Re}_L}} \qquad \overline{Nu}_L = \frac{\overline{h}L}{k_f} = 0.664\,\mathrm{Re}_L^{\,1/2}\,\mathrm{Pr}^{1/3}$$

- These results are valid as long as Pr is relative large, I.e. Pr ≥ 0.6.
- For very low Pr, like liquid metals, a better correlation is with the Peclet number:

$$Nu_x = 0.565\mathrm{Pe}_x^{\,1/2} \qquad \mathrm{Pr} \le 0.05 \qquad \mathrm{Pe}_x \ge 100$$



## Laminar Flow on Plates (cont)

- The book also provides an empirical result which applies over a full range of Pr's and Re's.
- However, let's consider our original result and consider the Reynold analogy. I.e:

$$\overline{\mathrm{St}} = \frac{\overline{\mathrm{Nu}}_L}{\mathrm{Re}_L\,\mathrm{Pr}} = f(C_f, \mathrm{Pr})$$

- From our equations, it is pretty obvious that the functional relation in this case is:

$$St = \frac{1}{2} C_f\,\mathrm{Pr}^{-2/3}$$



## Turbulent Flow on Plates

- No simple solution exists for turbulent flow and we must rely upon either CFD or experiment.
- For Reynolds numbers up to 10 Million, a good correlation for viscous flow is:

$$\delta = 0.381x\,\mathrm{Re}_x^{\,-1/5} \qquad C_f(x) = 0.0592\,\mathrm{Re}_x^{\,-1/5}$$

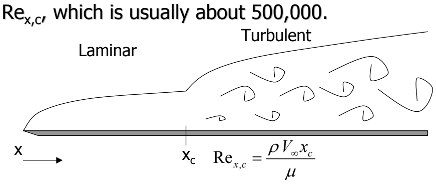
- Assuming the previous Reynolds analogy for laminar flow also applies to turbulent flow, then:

$$\mathrm{Nu}_x = \mathrm{St}\,\mathrm{Re}_x\,\mathrm{Pr} = 0.0296\,\mathrm{Re}_x^{\,4/5}\,\mathrm{Pr}^{1/3}$$

- Where this result is generally considered valid for Pr's ranging from 0.6 to 60.

## Mixed Flow on Plates

- However, for most plates, we must account for the existence of both laminar and turbulent flows.
- The transition from laminar to turbulence occurs at what is called the critical point, $x_c$.
- There is a corresponding critical Reynolds number, $Re_{x,c}$, which is usually about 500,000.






## Mixed Flow on Plates (cont)

- Assuming $Re_{x,c}$= 500,000 and averaging $C_f$ and Nu over both the laminar and turbulent ranges gives the correlations:

$$\overline{C}_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{1742}{\mathrm{Re}_L}$$

$$\overline{\mathrm{Nu}}_L = \left(0.037\,\mathrm{Re}_x^{4/5} - 871\right)\mathrm{Pr}^{1/3}$$

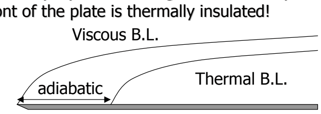
- Which are valid for:

$$500{,}000 \le \mathrm{Re}_L \le 10^8$$
$$0.6 \le \mathrm{Pr} \le 60$$



## Mixed Flow on Plates (cont)

- Finally, note that the textbook also gives solutions for some special cases including:
  - $L >> x_c$
  - Fixed wall heat flux rather than fixed wall temperature.
  - Cases where the thermal boundary layer and viscous boundary layer do not begin at the same point - I.e. the front of the plate is thermally insulated!




- When confronted with a new problem, take the time to find the best correlation that applies!

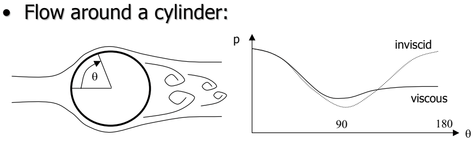
## Flow around Tubes

- The flow around bodies, and in particular around tubes, differs from flat plates in two ways:
  - The flow around bodies will have pressure gradients which effect the boundary layer development.
  - Some bodies have sharp corners which leads to flow separation.
- Pressure variations in a flow effect all aspects of the boundary layer including grow rate and transition location.
- In regions of rapidly increasing pressure, the boundary layer may even separate from the surface leaving a large region of recirculating flow as is the case for a cylinder:

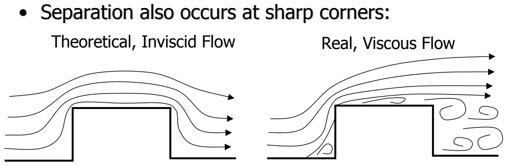


## Flow around Tubes (cont)

- Flow around a cylinder:




- Separation also occurs at sharp corners:






## Flow around Tubes (cont)

- In fact, the flow around a cylinder looks quite different depending upon the Reynolds number:
  - At very low $Re_D$, the entire flow is dominated by shear stresses and looks almost inviscid.
  - At low $Re_D$, a laminar separation will occur with a laminar wake.
  - At moderate $Re_D$, a laminar separation with a turbulent wake appears.
  - At higher $Re_D$, transition to turbulence occurs before separation.
  - At very high $Re_D$, the flow is entirely turbulent.
- As a result, our experimental correlations should account for this variability!

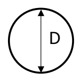
## Flow around Tubes (cont)

- For cross flow on tubes, experiment indicates that:

$$\overline{Nu}_D = \frac{\bar{h}D}{k_f} = C\,\mathrm{Re}_D^{\,m}\,\mathrm{Pr}^{1/3}$$

- Where the constants C and m depend upon geometry and $Re_D$.
- For circular tubes, Table 6-2 gives:

| $Re_D$ | C | m |
|---|---|---|
| 0.4 - 4 | 0.989 | 0.330 |
| 4 - 40 | 0.911 | 0.385 |
| 40 - 4000 | 0.683 | 0.466 |
| 4000 - 40,000 | 0.193 | 0.618 |
| 40,000 - 400,000 | 0.0266 | 0.805 |

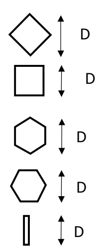





## Flow around Tubes (cont)

- For non circular tubes, Table 6-3 gives:

| $Re_D$ | C | m |
|---|---|---|
| $5 \times 10^3 - 10^5$ | 0.246 | 0.588 |
| $5 \times 10^3 - 10^5$ | 0.102 | 0.675 |
| $5 \times 10^3 - 2 \times 10^4$ | 0.160 | 0.638 |
| $2 \times 10^4 - 10^5$ | 0.0385 | 0.782 |
| $5 \times 10^3 - 10^5$ | 0.153 | 0.638 |
| $4 \times 10^3 - 1.5 \times 10^4$ | 0.228 | 0.731 |



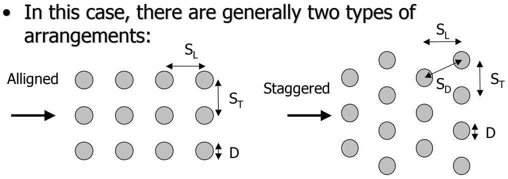



## Flow around Tubes (cont)

- The previous empirical relations should be valid for any flow with Pr > 0.6.
- The text also offers additional correlations for circular tubes which are either more recent or applicable for wider ranges of $Re_D$ and Pr.
- These equations are also slightly harder to use.
- However, given the fact that all of these correlations are accurate to about ±20%, it is generally better to use the simplest equation which works!.

## Flow Across Tube Banks

- There is one last external flow situation you should be familiar with - flow across tube banks.
- This situation occurs frequently in the design of heat exchangers, and we have already seen it applied to heat sinks for integrated circuits.
- In this case, there are generally two types of arrangements:






## Flow Across Tube Banks (cont)

- The heat transfer should be a function of not only the tube diameters, D, but also of:
  - Longitudinal pitch, $S_L$
  - Transverse pitch, $S_T$
  - Diagonal pitch, $S_D$ (for staggered arrangements)
  - Number of rows, $N_L$
- Experiment shows that a good correlation for $N_L > 10$ is:

$$\overline{Nu}_D = \frac{\bar{h} D}{k_f} = C \, \mathrm{Re}_{D,\max}{}^{n} \, \mathrm{Pr}^{1/3}$$

- Where the constants C and n are given by geometry and pitch spacing in Table 6-4.



## Flow Across Tube Banks (cont)

- But, note that this equation uses a different Reynolds number:

$$\mathrm{Re}_{D,\max} = \frac{\rho V_{\max} D}{\mu}$$

- Where $V_{max}$ is not the free stream flow, but the maximum velocity between the tubes.
- If we assume simple blockage and incompressible flow, than simple mass conservation would indicate that:

$$V_{\max} = \left(\frac{S_T}{S_T - D}\right) V$$

Alligned or Staggered

$$V_{\max} = \left(\frac{S_T / 2}{S_D - D}\right) V$$

Staggered if $S_T > 2S_D - D$

## Flow Across Tube Banks (cont)

- Finally, the book offers correction factors in Table 6-5 for cases where $N_L < 10$.
- One final complication is the fact that all the tubes do not see the same fluid temperature since it varies going downstream.
- To account for this, a log mean temperature is defined by:

$$\Delta T_{lm} = \frac{(T_s - T_i) - (T_s - T_o)}{\ln\left[(T_s - T_i)/(T_s - T_o)\right]}$$

- Where $T_i$ is the fluid temperature at the flow inlet and $T_o$ is the fluid temperature at the flow outlet.



## Flow Across Tube Banks (cont)

- An estimate for To may be obtained using:

$$\frac{(T_s - T_o)}{(T_s - T_i)} = \exp\left(-\frac{\pi \mathrm{D N \bar{h}}}{\rho \mathrm{V N_T S_T c_P}}\right)$$

- Where N is the total number of tubes and NT is the number of tubes in the transverse plane.
- Finally, the heat flux per unit length for the entire tube banks is found from:

$$\frac{q}{L} = N\bar{h}\,\pi\, D\Delta T_{lm}$$



## Flow Across Tube Banks (cont)

- Of course, the text book gives alternate correlations which can also be used.
- And, if you were designing a heat exchanger, the book also gives an estimation method to predict the pressure drop across the tube bank.
- This value is important since the pump or fan which moves the fluid will have to supply the equivalent pressure jump in order to keep the flow moving.