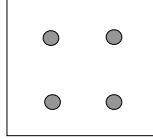


### Entropy

- To get a grasp on entropy, consider a collection of 4 particles in a box as shown.
- Next, imagine the energy each particle can carry is "quantized" into increments of 1 - I.e. each particle can only have energy levels of 1,2,3,4,....



- Now, how many ways can this system be arranged if the there is a total energy of 2? Now, how about if the total energy was 3?

ES 312 Energy Transfer Fund. 31 2/2/2007

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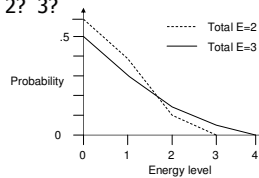
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### Entropy (cont)

- More importantly in terms of entropy, what is the probability of a particular particle having a particular energy level of 0? 1? 2? 3?
- If the probabilities are plotted versus energy level, the curve would look like this.
- The measure of randomness is the width, or standard deviation, of this curve!



ES 312 Energy Transfer Fund. 32 2/2/2007

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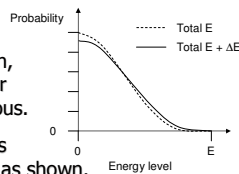
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### Entropy (cont)

- In a real system, energy is quantized but the energy increments is very, very small when compared to the total system energy.
- As a result, rather than a discrete energy distribution, the energy distribution is, for all useful purposes, continuous.
- The probability distribution is thus a Gaussian distribution as shown.
- However, the standard deviation (i.e., the curve width) still depends upon the total energy level!



ES 312 Energy Transfer Fund. 33 2/2/2007

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### Entropy (cont)

- We can now make a qualitative statement about entropy from our observations.
- The change of randomness in a system depends upon the ratio of two things:
  - Directly with the amount of energy added to the system
  - Inversely with the total energy already in the system.
- Qualitatively, we will express the energy added heat transfer,  $Q$ , and measure the energy already in the system with temperature,  $T$ .

• Thus, 
$$\Delta S = \frac{Q}{T} \quad \text{or} \quad dS = \frac{\delta Q}{T}$$

ES 312 Energy Transfer Fund. 34 2/2/2007

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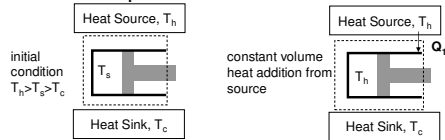
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### Cycles

- Now, to develop the 2nd Law of Thermo, we will apply this entropy relation to Cycles.
- A Cycle (or cyclic process) is a sequence of processes which return a system to its exact original state.
- For example consider the heat engine cycle as shown below in four processes:



ES 312 Energy Transfer Fund. 35 2/2/2007

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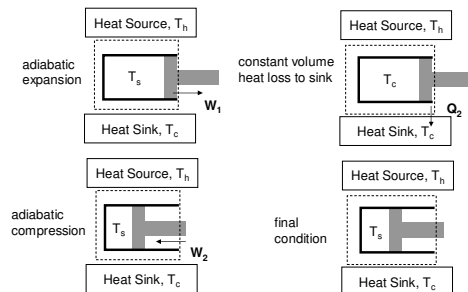
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### Cycles (cont)



ES 312 Energy Transfer Fund. 36 2/2/2007

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Cycles (cont)

- From the 1st Law of Thermo, if the total heat transfer and total work done are the sum of the individual processes,  $W = \sum W_i$  and  $Q = \sum Q_i$

$$\Delta E = \sum W_i + \sum Q_i = 0$$

- Where the total must equal zero since the initial and final states are the same:  $\Delta E = 0!$
- To find the total work done, we then need to calculate the total heat transfer! In steps:

$$Q_1 = \Delta E = \Delta U = mc_v(T_h - T_s)$$

$$Q_2 = \Delta E = \Delta U = mc_v(T_s - T_c)$$

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Cycles (cont)

- Thus the total work is:

$$W = -\sum Q_i = -Q_1 - Q_2$$

$$= -mc_v(T_h - T_s) - mc_v(T_s - T_c)$$

- Or,

$$W = -mc_v(T_h - T_c)$$

- Thus, in this cycle, a net work is done by the system on the surroundings.
- Note that the entire process can also be reversed to refrigerate!

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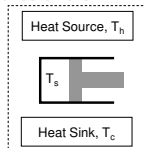
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The 2nd Law of Thermo.

- Now that we have the concept of entropy and of cycles, lets see to relate them.
- Let's revisit the engine cycle we looked at, but change the C.M. to include the source and sink reservoirs.

- Now consider the transfer of heat from the source to the piston cylinder. Particularly, calculate the entropy change associated with this process.




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The 2nd Law of Thermo. (cont)

- The heat transfer,  $Q_1 = mc_v(T_H - T_S)$ , is internal, but the total entropy change would be the sum of the entropy changes of the reservoir and the piston-cylinder.

$$\Delta S = \int dS = \int \frac{\delta Q}{T} = \int_{source} \frac{\delta Q}{T} + \int_{piston/cyl} \frac{\delta Q}{T}$$

- For the large reservoir, the temperature remains constant at  $T_H$  but  $\delta Q$  is negative, i.e.  $\delta Q_{piston/cyl} = -\delta Q_{source}$ .

- Thus, combining the two integrals:

$$\Delta S = \int \left( \frac{-1}{T_H} + \frac{1}{T} \right) \delta Q = \int \left( \frac{T_H - T}{TT_H} \right) \delta Q$$

ES 312 Energy Transfer Fund. 40 2/2/2007

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The 2nd Law of Thermo. (cont)

- Thus, since  $T_H$  is greater than  $T$ , there will be a net increase in entropy. A similar result can be obtained for the second heat transfer process.
- What is interesting is that this happens without anything being done to the system!
  - No heat has been added
  - No work has been done
- Thus, the total system energy has not changed, but its entropy has - what does this mean?
- Also, note that the only way not to have an entropy increase is if  $T_H = T_S$  i.e. no temperature difference.

ES 312 Energy Transfer Fund. 41 2/2/2007

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The 2nd Law of Thermo. (cont)

- This result leads us to one way to state the 2nd Law of Thermo:

$$\Delta S_{everything} \geq 0$$

- This result was obtained with a little slight of hand, but its result is valid: for every process, the entropy of the universe remains the same or increases.
- Entropy increases if the process involves a non-equilibrium process such as heat transfer, friction, diffusion.
- The entropy change can be made quite small (or zero) if the level of non-equilibrium is kept small - like if  $T_H \approx T_S$ .

ES 312 Energy Transfer Fund. 42 2/2/2007

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### The 2nd Law of Thermo. (cont)

- A reversible process can now be defined more precisely - it is one in which

$$\Delta S_{\text{everything}} = 0 \quad \text{Reversible Process}$$

- By necessity, this would be a process without heat transfer or friction.
- Practically, this rule is satisfied if the temperature gradients are small and friction is negligible.
- When a process is irreversible, some of the original energy is made unusable due to work - it often shows up in the form of waste heat.

ES 312 Energy Transfer Fund. 43 2/2/2007

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### The 2nd Law of Thermo. (cont)

- If we don't want to include the entire universe, another way to state the 2nd Law is:

$$dS = \delta Q/T + dS_{\text{generation}} \quad dS_{\text{generation}} \geq 0$$

- This equation states that entropy varies as heat is added or removed, but if done irreversibly, a slight entropy generation (always positive) also occurs.
- Predicting the amount of irreversible entropy generation is sometimes possible, but not always since details of the process are necessary.
- In practice, the level of irreversibility,  $dS_{\text{generation}}$  is one measure of process efficiency.

ES 312 Energy Transfer Fund. 44 2/2/2007

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### Isentropic Processes

- A flow which is adiabatic **and** reversible is called **isentropic**.

- Literally, this means the entropy of the system,  $S$ , is a constant.
- Practically, this means that some special relations exist between our fluid properties.

- To see this, start with the 1st Law equations:

$$\delta Q = dU + p dV \quad \delta Q = [dH + d(pV)] + p dV$$

$$= dH - V dp$$

- include the definitions of specific heats:
- $$0 = mc_v dT + p dV \quad 0 = mc_p dT - V dp$$

ES 312 Energy Transfer Fund. 45 2/2/2007

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### Isentropic Processes (cont.)

- rearrange and divide

$$\frac{c_p dT}{c_v dT} = \gamma = \frac{V dp}{-p dV} \quad \text{or} \quad \frac{dp}{p} = -\gamma \frac{dV}{V} = -\gamma \frac{dv}{v}$$

• where  $\gamma = c_p/c_v$  = ratio of specific heats

- Now, integrate over the change from one condition, 1, to another, 2

$$\ln\left(\frac{p_2}{p_1}\right) = -\gamma \ln\left(\frac{v_2}{v_1}\right)$$

$$\text{or} \quad \left(\frac{p_2}{p_1}\right) = \left(\frac{v_2}{v_1}\right)^{-\gamma} = \left(\frac{\rho_2}{\rho_1}\right)^\gamma$$

ES 312 Energy Transfer Fund.

46

2/2/2007

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### Isentropic Processes (cont.)

- We can also use the perfect gas law to introduce T

$$\left(\frac{p_2}{p_1}\right) = \left(\frac{\rho_2}{\rho_1}\right)^\gamma = \left(\frac{p_2}{RT_2} \frac{RT_1}{p_1}\right)^\gamma$$

$$\text{or} \quad \left(\frac{p_2}{p_1}\right) = \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)}$$

- To summarize, for isentropic flow,  $r$ ,  $p$  and  $T$  are related by

$$\left(\frac{p_2}{p_1}\right) = \left(\frac{\rho_2}{\rho_1}\right)^\gamma = \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)}$$

ES 312 Energy Transfer Fund.

47

2/2/2007

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