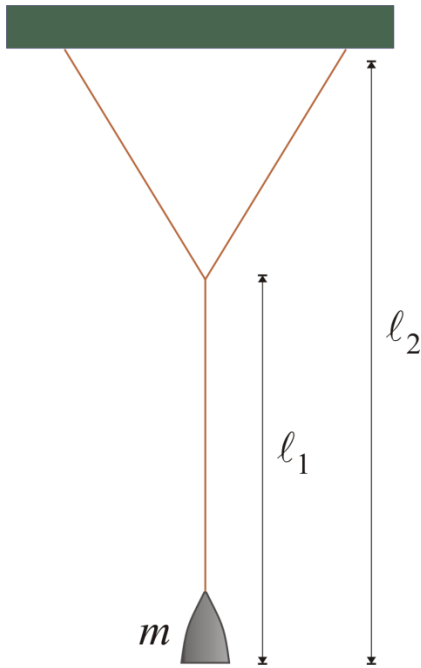


**PART I**

Suspend a small weight from thin string or thread to make a “Y-pendulum” as shown in the figure below. The ratio of the length  $\ell_2$  to the length  $\ell_1$  (see figure below) should be 4 to within about 5%. Make sure that the line between your two support points is perpendicular to the vertical and that the two pieces of thread making up the two arms of the “Y” are of equal length. The longer your pendulum, the longer the periods and the easier this exercise will be to carry out. So, I recommend suspending the pendulum from a top bunk, ceiling, light fixture, etc.



After constructing and suspending your pendulum, please take a photograph of it and submit the photograph with this homework.

Before going any further, we'll define a set of orthogonal axes. Let the origin be at the equilibrium position of the bob and let the  $z$ -axis point up (along the equilibrium position of the string). Let the  $x$ -axis point to the right in the image to the left, i.e. parallel to the support points. Let the  $y$ -axis point into the paper in the image to the left, i.e. perpendicular to the plane of the “Y” formed by the thread.

Your pendulum should now oscillate with two different periods, depending on whether it was released from a position immediately above the  $x$ -axis (oscillations in the plane of the “Y”) or from a position immediately above the  $y$ -axis (oscillations in the plane perpendicular to the plane of the “Y”). Measure and record the two periods of oscillation using a watch or clock recording seconds to time 10 full, back-and-forth, swings of the pendulum (and then divide by ten). Remember that a pendulum is not a simple harmonic oscillator except for small displacements, so try to keep the amplitude of the swings under about 30 degrees. The  $y$ -axis period should be longer than the  $x$ -axis period by a factor of 2. If it is not, then adjust the

lengths  $\ell_1$  and  $\ell_2$  until you obtain the correct ratio. You are then ready to proceed with steps 1-3 below.

1. Displace the pendulum bob by approximately equal amounts in the  $x$  and  $y$  directions and release it from rest. Sketch the trajectory that the bob traces out in the  $xy$ -plane. Does the pattern repeat itself?
2. Displace the pendulum bob in the  $y$ -direction only ( $x = 0$ ) but give the bob a non-zero velocity in the  $x$ -direction ( $v_x \neq 0$ ) so that the  $x$  and  $y$  amplitudes end up being approximately the same. Be careful not to give the bob any initial  $y$ -component of velocity. Sketch the trajectory that the bob traces out in the  $xy$ -plane. Does the pattern repeat itself?
3. Repeat Step 2 but interchange  $x$  and  $y$ .

Submit your sketches with this homework.

**PART II**

Since the pendulum is a simple harmonic oscillator for small displacements, the motion in the  $x$  and  $y$  planes are both described by sinusoidal functions of time. In other words,  $x(t) = x_0 \cos(\omega_x t + \phi_x)$  and

$y(t) = y_0 \cos(\omega_y t + \phi_y)$ . By choosing  $\omega_x$ ,  $\omega_y$  and  $\phi_x$ ,  $\phi_y$  appropriately, the trajectories in steps 1-3 above can be reproduced with a parametric plot of  $x(t)$  versus  $y(t)$ . Use Matlab to plot the trajectory that the bob traces out in each of steps 1-3 above. Plot the traces together in a single plot window (on a single set of axes) using different colors and line styles for each of the three trajectories. Print your plot and submit with this homework.