Show all your work and submit all plots. Also, print out, label, and submit any code.

1. A mass suspended from a damped spring can be driven by moving the top of the spring (i.e. the suspension point) sinusoidally with frequency f_d. The resulting motion is governed by the equation for the driven damped simple harmonic oscillator

$$\ddot{x} + \beta\dot{x} + \omega_0^2 x = \omega_0^2 x_d \cos(\omega_d t)$$

where β is the damping factor, $\omega_0 = \sqrt{k/m}$ is the resonant frequency of the corresponding free undamped simple harmonic oscillator, $x_d \cos(\omega_d t)$ is the position of the motion of the top of the spring as a function of time where $\omega_d = 2\pi f_d$ and m is the mass of the object being driven. In parts a.-f. below, let $\beta = 10\,\mathrm{s}^{-1}$, k =90 N/m, m =0.1 kg, and $x_d = 0.01$m. The steady state solution to this differential equation is: $x(t) = x_0 \cos(\omega t + \phi)$, where x_0 and ϕ are functions of ω.

   a. Derive expressions for ω, x_0/x_d and ϕ in terms of the parameters and the angular frequency ω_d at which the oscillator is driven. Define a vector (1xN matrix) of frequencies $f_d = \omega_d/2\pi$, starting at 1 Hz and ending at 100Hz, that is exponentially spaced. (In other words log(f_d) is linearly spaced. That is, the difference between consecutive elements of log(f_d) is uniform). The Matlab function "logspace" should help greatly. Using the exponentially spaced vector you generated, plot x_0 versus f_d with f_d on the abscissa ("*x*"-axis) assuming the values for the physical parameters given above. In a separate set of axes plot ϕ vs. f_d.

   b. Now, plot log10(x_0/x_d) versus log10(f_d) with log(f_d) on the abscissa using the usual plotting command "plot". This type of plot is known as a *log plot* or sometimes a *log-log plot*. Then, on a separate set of axes use the command "loglog" to plot x_0/x_d versus f_d on logarithmically "ruled" axes with f_d on the abscissa. How do the two plots compare? (You may find the command "grid on" to be useful for visualizing how the axes are ruled.)

   c. Find the asymptotic form of the function $x_0(f_d)/x_d$ as $f_d \to \infty$.

   d. Re-plot log10(x_0/x_d)versus log10(f_d) with log10(f_d) on the abscissa for a wider frequency range: 1 Hz-1MHz. In the limit of large f_d, you should see that the log-log plot above becomes a <u>straight line</u> with a slope of -2. By appealing to part c. above, explain both the fact that the curve becomes a straight line for large f_d and the fact that the slope is -2.

   e. Use the command "ax=plotyy(… , … , … , … , 'loglog' , 'semilogx')" to plot both x_0/x_d versus and the phase lag ϕ versus f_d on the same graph (with log(f_d) on the abscissa). You will of course need to fill in the …'s appropriately in the command above. Use f_d in the range 1 Hz – 100 Hz. You may want to read the Matlab Help for plotyy. Figure out how to label the left hand *y*-axis with the string "Amplitude Response (m)", and the *x*-axis with the string "Frequency (Hz)".

   f. Use "set(ax(2),'ytick',[-360:90:360])" to change the right hand *y*-axis tick locations from the default to something that is more meaningful when the axis represents degrees. To understand this command, you will need to read Sections 31.1 and 32.2 in "Basics of Matlab and Beyond." Now use a similar command to label the right-hand *y*-axis with the string "Phase Lag (degrees)". You may want to read the Matlab Help for "plotyy" and section 32.5 in "Basics of Matlab and Beyond."