Homework 5 PS232

Show all your work and submit all plots. Also, print out, label, and submit any code.

1. Modify the routine "decay.m" so that the integration is done with:
   a. the midpoint method
   b. the fourth order Runge-Kutta method

   On the same set of axes, plot the numerical solution for each method along with the exact solution. (Do not change the size of the time step *h* between methods.)

Notes for problems 2 and 3. It is quite easy to use Euler's method to estimate the solution to second order equations such as those arising from Newton's 2nd Law. In that case, we have an expression, not for the time derivative of position (velocity), but for the time derivative of velocity (i.e. acceleration). But, by applying Euler's method to first get an estimate for the velocity from the acceleration, we can then re-apply Euler's method a second time to get an estimate of position from the velocity.

2. For this problem refer to the numerical solution for the damped harmonic oscillator in the file "dsho.m" which uses Euler's Method. Construct a plot of the GDE versus time step *h*. Choose your time step vector to be "*h*=logspace(-5,-3,20)". [Hint: You will need run the code for each time step value *h(n)* and record the maximum deviation of the numerical solution from the exact solution for each value of *h(n)*.]

3. The Matlab routine "newtongrav_earth_moon.m" on the course website implements a numerical integration of Newton's Universal Law of Gravitation to estimate the orbits of the earth and moon about one another. (Note that on a very slow computer, this simulation won't run very well and you will need to decrease the value of tend and increase the value of dt.)

   a. *Without changing the initial* conditions or the end time "tend", increase the time-step "dt" by a factor of 10. Let's call the original time step dt1 and the new, larger time step dt2. The matrix variable "r" holds the distance from the origin of the earth and the moon as a function of the time vector, t. (Column 1 of r corresponds to the earth and column 2 to the moon). Call the "r" that is generated with time step dt1, "r1" and call the "r" that is generated with time step dt2, "r2". Similarly, call the respective time vectors "t1" and "t2". Since dt2 is larger than dt1 by a factor of 10, r2 should have ten times fewer rows than r1. Use the function "interp1" to interpolate r2 over the time vector t1 so that it is the same size as r1. Then plot t1 versus (r2new-r1) where r2new is the interpolated version of r2. Note how the errors due to the larger time step grow with time. On average, does the error grow approximately linearly with time?

   b. Add a third body to the problem along with appropriate initial conditions. Use this code to model the motion of a spacecraft orbiting Earth's moon. Choose appropriate values for the spacecraft mass and initial conditions. Try to obtain stable orbits of the spacecraft around the moon (as the moon orbits the earth) when:
      i. All three bodies are in the same plane.
      ii. The spacecraft does not orbit in the same plane as the earth and moon.
      (To investigate the 3D nature of the orbit you can use the plot "Rotate 3D" widget in the figure toolbar.)