Stability and Control

- Stability & control is the science behind keeping the aircraft pointed in a desired direction. Whereas performance analysis sums the forces on aircraft, stability & control analysis requires summing the moments acting on it as a result of surface pressure and shear-stress distributions, engine thrust, etc., and ensuring those moments sum to zero when the aircraft is oriented as desired.
- Stability analysis: deals with the changes in moments on the aircraft when it is disturbed from equilibrium (the condition when all forces and moments on it sum to zero).
- Control analysis: determines how the aircraft should be designed so that efficient control authority is available to allow the aircraft to fly all maneuvers and at all speed required by the design specifications.

Coordinate System of Positive Moment Directions

- A natural starting point for the coordinate system: center of gravity (CG), where aircraft rotates about it. This is called the body axis system.
- Longitudinal axis (x): aircraft’s center line is chosen parallel to and usually coincident with its aircraft reference line (a line drawn down the fuselage from nose to tail). Positive toward the nose. Aircraft rotation about the longitudinal axis (roll) to the right wing down is positive. The rolling moment is labeled: $\mathcal{L}$ (to avoid confusion against lift).
- Vertical axis (z): in order for the right-hand-rule to be satisfied, the vertical axis must be set positive downward. Aircraft rotation about the vertical axis (yaw) to the right (clockwise) is positive. The yawing moment is labeled: $N$.
- Lateral axis (y): positive out the right wing. Aircraft rotation about the lateral axis (pitch) up is positive. The pitching moment is labeled: $M$.
Degrees of Freedom (DOF)

- The aircraft has six degrees of freedom (DOF), six ways it can move.
- Three DOF in translation (linear motion), which are orthogonal to each other. The three components of force along x, y, and z axes are labeled: X, Y, and Z. The three components of velocity along x, y, and z axes are labeled: u, v, and w.
- Three DOF in rotation, which are also orthogonal to each other: The three components of moment along x, y, and z axes are labeled: L (roll), M (pitch), and N (yaw).

Control Surfaces and Rotation

- **Rolling** about longitudinal (x) axis: control surfaces on the aircraft's wings called ailerons deflect differentially (one trailing edge up and the other down) to create more lift on one wing, less on the other, and therefore a net rolling moment is generated.
- **Pitching** about lateral (y) axis: a control surface near the rear of the aircraft called an elevator or stabilator is deflected so that it generates a lift force, from which (because of its moment arm from the aircraft’s center of gravity) creates a pitching moment. An elevator is a movable surface attached to a fixed horizontal surface (stabilizer). A stabilator is a combined stabilizer + elevator ("all moving" surface). Left & right stabilators may move opposite each other to provide roll control.
- **Yawing** about vertical (z) axis: a movable surface called a rudder, which is attached to the aircraft's fixed vertical stabilizer, deflects to generate a lift force in a sideways direction for yaw control.

Other Control Surfaces

- **Flaperons** (flap + aileron): F-16 in fig. 6.2a combines the functions of an aileron with a plain flap.
- **Canards**: stabilator placed forward of the main wing.
- **Elevons** (elevator + aileron) for tailless aircraft: move together to function as elevators to provide pitch control or move differentially like ailerons to provide roll control.
- **Ruddervators** (rudder + elevator) for V-tail aircraft: function as elevators when moving together or function as a rudder when moving differentially.
Trim

- When the sum of the moments about an aircraft's center of gravity is zero, the aircraft is said to be **trimmed**. The act of adjusting the control surfaces of an aircraft so that they generate just enough force to make the sum of the moments zero is called **trimming** the aircraft.
- The control surfaces that deflect to create the needed forces and moments to trim the aircraft are sometimes called **trimming surfaces**.
- The trimming condition is an equilibrium condition in terms of **moments** (not necessarily forces).
- An aircraft in a steady, level turn would be considered "trimmed" if the sum of the moments acting on it is zero. An aircraft in a level, straight ahead acceleration would still be trimmed in order to remain level, even though the forces generated by trimming surfaces to keep the sum of the moments zero would likely need to change as the aircraft’s speed changed.

Static & Dynamic Stability

- Stability is the tendency of a system, which disturbed from an equilibrium condition, to return to that condition.
- **Static stability**: the initial tendency or response of a system when it is disturbed from equilibrium. If the initial response of the system, when disturbed, is to move back toward equilibrium, then the system is said to have positive static stability.
- **Dynamic stability**: the response of the system over time. The history of a system (oscillation amplitude) is diminishing (damping) toward equilibrium is said to have positive dynamic stability.
The analysis of the problem of adjusting pitch control to change (or stabilize) the aircraft's pitch attitude is called longitudinal control analysis (because the moment arms for the pitch control surfaces are primarily distances along the aircraft's longitudinal axis). Also, the conditions required for longitudinal trim are affected by the airplane's velocity (primarily in the longitudinal direction).

A longitudinal problem involves two degrees of translational freedom ($x$ and $z$ directions) and one degree of freedom in rotation (about $y$ axis). The analysis determines the lift required from trimming surfaces to reduce the net moment on the aircraft to zero, while at the same time, keeping the net forces on the aircraft zero.

### Longitudinal Trim

1. Fig. 6.6 illustrates the longitudinal trim problem for a conventional tail-aft airplane. Summing the moments about the aircraft's center of gravity yields:
   \[ \sum M_{cg} = M_{ac} + L(x_{cg} - x_{ac}) - L_vl \quad (6.1) \]

2. The moments in eq(6.1) must sum to zero, if aircraft is trimmed. For steady flight, the forces must also sum to zero. Summing forces in the vertical direction yields:
   \[ \sum F_L = 0 = L + L_v - W \quad (6.2) \]

### Control Authority

1. If an aircraft's geometry and flight conditions are known, then the lift coefficient required from the wing and pitch control surfaces can be determined using $L = C_L q S$ when eq(6.1) and eq(6.2) are solved for $L$ and $L_v$. If any of the required $C_L$ values are greater than $C_{L_{max}}$ for their respective surfaces, then the aircraft does not have sufficient control authority to trim in that maneuver for those conditions.

2. To remedy this situation, the aircraft designer must either increase the size of the deficient control surface or add high-lift devices to it to increase its $C_{L_{max}}$. Fig. 6.7 shows F-4E with stabilator leading-edge slots added to increase $C_{L_{max}}$ to secure control authority.
Example D-1-1
(Longitudinal Control Analysis)

Example 6.1
A design concept for a light general aviation aircraft uses a canard configuration as shown in Fig. 6.8. Both the wing and the canard of this aircraft have rectangular planforms. The aircraft has a mass of 1,500 kg and is designed to fly as slow as 30 m/s at sea level in level flight. At this speed, its cambered main wing generates −1,000 N-m of pitching moment about its aerodynamic center. If the maximum lift coefficient for its canard is 1.5, how large must the canard be in order to trim the aircraft at its minimum speed?

Solution (6.1)
Longitudinal Static Stability

Static Stability Criterion

- A criterion for positive longitudinal stability: a condition that must be satisfied in order for an aircraft to be stable: pitching moment must decrease with increasing angle of attack and increase with decreasing angle of attack.

\[
\frac{\partial C_{M_{eq}}}{\partial \alpha} = C_{M_\alpha} < 0 \quad (6.3) \]  

Longitudinal static stability criterion

Trim Diagram

- A plot of pitching-moment coefficient v.s. angle of attack (or lift coefficient) reveals the relationship between static stability and trim, and usually called trim diagram.
- \( C_{M_{eq}} \) v.s. \( \alpha \) curve slope, \( C_{M_\alpha} \) is constant (typical at low angles of attack).
- The coefficient where \( C_{M_{eq}} = 0 \) (and \( C_L = 0 \)) is given the symbol \( C_{M_0} \) (called, moment coefficient at zero lift).
- The angle of attack where \( C_{M_{eq}} = 0 \) is called the trim angle of attack (\( \alpha_{trim} \)).
- Fig. 6.10 immediately makes obvious another requirement for an aircraft with positive longitudinal stability: because \( C_{M_\alpha} < 0 \) for stability, it follows that \( C_{M_0} \) must be greater than 0, if the aircraft is to trim at a useful \( \alpha_{trim} \) (required characteristic of aircraft that have positive static longitudinal stability and useful trim angles of attack).
Neutral Point & Static Margin

Neutral Point

- Aerodynamic center of an airfoil is a point on the airfoil about which the net aerodynamic moment does not change with angle of attack.
- Likewise, a similar point can be found on an aircraft where net total moment on the aircraft does not change with angle of attack. The total lift can be placed at this point, called the neutral point, to analyze the static longitudinal stability of an aircraft.
- The name "neutral point" is because placing its center of gravity there gives neutral static stability.
- If the aircraft's angle of attack increases, its lift will increase. In fig. 6.12 pitching up (increased total lift) will generate a negative (nose-down) moment about its center of gravity: positive static longitudinal stability.
- In fig. 6.13, if the aircraft's center of gravity moves aft, an increase in angle of attack and the resulting increase in lift would cause a positive (nose-up) pitching moment about its center of gravity: negative static longitudinal stability.

Static Margin (SM)

- The distance of an aircraft's center of gravity ahead of its neutral point, when divided by its reference chord length, is called its static margin (SM). If an aircraft's SM is positive, it is stable.
- Stated in terms of SM, the static longitudinal stability criterion becomes SM > 0.
- A large SM suggests an aircraft that is very stable and not very maneuverable aircraft. A low positive SM is normally associated with highly maneuverable aircraft. Aircraft with zero or negative SM normally require a computer fly-by-wire control system, in order to be safe to fly.
Altering Longitudinal Static Stability

- Simplification (1): aircraft reference line is the line such that when it is aligned with the freestream velocity, the wing and fuselage together produce zero lift.
- Simplification (2): contribution of the horizontal tail lift to the whole aircraft lift (but not the tail's contribution to the moment) will be ignored.
- With these assumptions, $L = 0$ and $\alpha_a = \alpha$.
- At the horizontal stabilizer, the local flow velocity vector is the vector sum of the freestream velocity and the downwash velocity $V_t$. The downwash angle is $\epsilon$. The angle of attack of the horizontal tail (stabilator) is labeled $\alpha_t$. The tail incidence angle $i$ is the angle between the horizontal tail chord line and the aircraft reference line.
- The expression for $C_{M_0}$ can be obtained by dividing eq(6.1) by $qSc$, where $c$ is the reference chord length of the wing:

$$
\frac{\sum M_{cg}}{qSc} = \frac{M_{ac} + L \left( x_{cg} - x_{ac} \right) - L_t}{qSc} = C_{M_{cg}} = C_{M_{ac}} + C_L \left( \frac{x_{cg} - x_{ac}}{c} \right) - \frac{C_{t_v} qS_i}{qSc} (6.4)
$$

$$
\bar{x}_{cg} = \frac{x_{cg}}{c}, \quad \bar{x}_{ac} = \frac{x_{ac}}{c}, \quad V_H = \frac{S_i}{Sc} (6.5) \Rightarrow C_{M_{cg}} = C_{M_{ac}} + C_L \left( \bar{x}_{cg} - \bar{x}_{ac} \right) - C_{t_v} V_H (6.6)
$$

$V_H$: horizontal tail volume ratio

- The lift coefficients of the wing and horizontal tail are:

$$
C_L = C_{L_v} (\alpha - \alpha_{L=0}) = C_{L_v} \alpha_a \quad \text{and} \quad C_{t_v} = C_{t_v} (\alpha_t - \alpha_{L=0})
$$

(horizontal tail is assumed to have a symmetrical airfoil): $\alpha_{L=0} = 0$, also: $\alpha_t = \alpha_a - \epsilon - i_t$

$$
C_{M_{cg}} = C_{M_{ac}} + C_{L_v} \alpha_a (\bar{x}_{cg} - \bar{x}_{ac}) - C_{t_v} (\alpha_a - \epsilon - i_t) V_H (6.7)
$$
Altering Longitudinal Static Stability (Continued)

- Now, $C_{M_0}$ is defined as the moment coefficient when the entire aircraft produces zero lift. For most airplanes the wing and fuselage together produce a very large portion of the lift, and so the lift of the tail has been neglected in this analysis.

\[
C_{M_0} = C_{M_{ac}} - C_{t_{ta}} (\varepsilon - i_t) V_H \Rightarrow C_{M_0} = C_{M_{ac}} + C_{t_{ta}} (\varepsilon_0 + i_t) V_H \quad (6.8)
\]

$\varepsilon_0$: downwash angle (when $\alpha = 0$, this is very small: $\approx 0$)

- The moment-curve slope $C_{M_a}$ is obtained by taking the derivative of eq(6.7) with respect to $\alpha_a$:

\[
C_{M_a} = \frac{\partial C_{M_{ac}}}{\partial \alpha_a} = \frac{\partial}{\partial \alpha_a} \left[ C_{M_{ac}} + C_{t_{ta}} \alpha_a (\bar{x}_{cg} - \bar{x}_{ac}) - C_{t_{ta}} (\alpha_a - \varepsilon - i_t) V_H \right] \quad (6.9)
\]

\[
\Rightarrow C_{M_a} = C_{t_{ta}} (\bar{x}_{cg} - \bar{x}_{ac}) - C_{t_{ta}} \left(1 - \frac{\partial \varepsilon}{\partial \alpha} \right) V_H \quad (6.10)
\]

- Eq(6.8) reveals that, because $C_{M_{ac}}$ is normally negative or zero and $\varepsilon_0$ is normally very small, the incidence angle of the horizontal tail must NOT be zero if $C_{M_0}$ is to be positive.

- $i_t$ was defined as positive when the horizontal tail is oriented so that it is at a lower angle of attack than the horizontal tail is oriented; hence, it is at a lower angle of attack than the main wing. When the main wing is producing no lift, the tail, if $i_t > 0$, will be at a negative angle of attack. The lift produced by the tail in this situation would be downward, creating a nose-up pitching moment, so that $C_{M_0} > 0$. 

Example 6.2

A conventional tail-aft flying model aircraft has the characteristics shown. What is the aircraft’s trim speed (the speed at which it will fly in equilibrium) at sea level? What would happen if the aircraft were launched at 15 ft/s?

<table>
<thead>
<tr>
<th>Wing</th>
<th>Tail</th>
<th>Airplane</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S = 0.8 \text{ ft}^2$</td>
<td>$S_{\text{tail}} = 0.333 \text{ ft}^2$</td>
<td>$x_{\text{sc}} = 0.1 \text{ ft}$</td>
</tr>
<tr>
<td>$c = 0.4 \text{ ft}$</td>
<td>$c_{\text{tail}} = 0.333 \text{ ft}$</td>
<td>$x_{\text{cg}} = 0.2 \text{ ft}$</td>
</tr>
<tr>
<td>$C_{L_a} = 0.078/\text{deg}$</td>
<td>$C_{L_{\text{nat}}} = 0.068/\text{deg}$</td>
<td>$I_t = 1.2 \text{ ft}$</td>
</tr>
<tr>
<td>$\delta 0 = 0$</td>
<td>$\delta_t = 4.8 \text{ deg}$</td>
<td>$C_{M_{\text{mb}}} = -0.04$</td>
</tr>
<tr>
<td>$\delta e/\delta \alpha = 0.2$</td>
<td></td>
<td>$W = 0.03 \text{ lb}$</td>
</tr>
</tbody>
</table>

Solution (6.2)
Mean Aerodynamic Chord (MAC)

- The reference wing chord length $\overline{c}$ is used in the expression for moment coefficient. The most commonly used value for $\overline{c}$ is known as the mean aerodynamic chord (MAC).

\[
MAC = \frac{1}{S} \int_{-b/2}^{b/2} c^2 \, dy \quad (6.11)
\]

- For untampered wings, $MAC = c$. For linearly tapered wings:

\[
MAC = \frac{2}{3} \frac{c_{\text{root}}}{1 + \lambda} \left(1 + \lambda^2 \right) \quad (6.12)
\]

- Note that $\lambda$ is a taper ratio: $\lambda = c_i/c$.  (4.1)

Aerodynamic Center of the Wing

- The advantage of using MAC for $\overline{c}$ is that it not only is used in defining moment coefficient, but it also can be used to approximate the location of the wing’s aerodynamic center.

- Just as the aerodynamic center of airfoil is normally located at about 0.25$c$, for wings the aerodynamic center is located approximately at 0.25$c$ of the MAC for Mach numbers below $M_{\text{crit}}$.

- At supersonic speeds, the aerodynamic center shifts to approximately 0.5 MAC. For swept wings, the spanwise location of the MAC is important because it must be known in order to locate the wing aerodynamic center.

- For untampered or linearly tapered wings, the spanwise location of the MAC $(y_{\text{MAC}})$ is given by:

\[
y_{\text{MAC}} = \frac{b(1+2\lambda)}{6(1+\lambda)} \quad (6.13)
\]

- The aerodynamic center of swept wings is, then:

$x_{ac} = y_{\text{MAC}} \tan \Lambda_{\text{LE}} + 0.25 \text{ MAC} \quad (6.14)$ for subsonic flight

$x_{ac} = y_{\text{MAC}} \tan \Lambda_{\text{LE}} + 0.5 \text{ MAC} \quad (6.15)$ for supersonic flight
Neutral Point Location & Static Margin

\[ x_{ac_{wing+strake}} = \frac{x_{ac_{wing}} S + (x_{ac_{strake}} - x_{ac_{wing}}) S_{strake}}{S + S_{strake}} \]  \hspace{1cm} (6.16)

\[ l_f w_f^2 \left[ 0.005 + 0.111 \left( \frac{l_{ac_{wing+strake}}}{l_f} \right)^2 \right] \]  \hspace{1cm} (6.17)

\[ C_{n_{a}} = 0 = C_{L_{0}}(\bar{x}_{cg} - \bar{x}_{ac}) - C_{L_{0}} \left( 1 - \frac{\partial E}{\partial \alpha} \right) V_H \]  \hspace{1cm} (6.18)

\[ \bar{x}_{cg}(\text{for } C_{n_{a}} = 0) = \bar{x}_{n} = \bar{x}_{ac} + V_H \frac{C_{L_{0}}}{C_{L_{0}}} \left( 1 - \frac{\partial E}{\partial \alpha} \right) \]

\[ \text{SM} = \bar{x}_{n} - \bar{x}_{cg} \]  \hspace{1cm} (6.19)

\[ C_{M_{x}} = -C_{L_{0}} \text{(SM)} \]  \hspace{1cm} (6.20)

Fuselage and Strake Effects

- Strakes or leading-edge extensions (LEXs), as well as fuselages, tend to shift the aerodynamic center so that the location of the aerodynamic center of the wing/fuselage combination is not the same as for the wing alone.
- The effect of strakes and leading-edge extension can be estimated by treating them as additional wing panels, using eq(6.14) to locate the aerodynamic center of the strake by itself, and then calculating a weighted average aerodynamic center location, with the areas of the strake and wing providing the weight factor:

\[ \frac{x_{ac_{wing+strake}}}{S + S_{strake}} = \frac{x_{ac_{wing}} S + (x_{ac_{strake}} - x_{ac_{wing}}) S_{strake}}{S + S_{strake}} \]  \hspace{1cm} (6.16)

- The effect of the fuselage on the aerodynamic center is approximated using an expression obtained from extensive wind-tunnel testing:

\[ l_f w_f^2 \left[ 0.005 + 0.111 \left( \frac{l_{ac_{wing+strake}}}{l_f} \right)^2 \right] \]  \hspace{1cm} (6.17)

- Moving the aerodynamic center forward is destabilizing. As a result, increasing the size of an aircraft’s fuselage and/or strakes would reduce a commensurate increase in the size of the horizontal tail, if the same aircraft stability is to be maintained.

Calculating Neutral Point Location and Static Margin (SM)

- Recall that the neutral point is the location of the center of gravity that would cause the airplane to have neutral static longitudinal stability \( (C_{M_{x}} = 0) \):

\[ C_{M_{x}} = 0 = C_{L_{0}}(\bar{x}_{cg} - \bar{x}_{ac}) - C_{L_{0}} \left( 1 - \frac{\partial E}{\partial \alpha} \right) V_H \]  \hspace{1cm} (6.18)

- For positive static longitudinal stability, need positive static margin (SM): \( \text{SM} = \bar{x}_{n} - \bar{x}_{cg} \) \hspace{1cm} (6.19)

- Relationship between SM, with lift/moment-curve slope is: \( C_{M_{x}} = -C_{L_{0}} \text{(SM)} \) \hspace{1cm} (6.20)
A design concept for a jet close-air-support aircraft uses a configuration as shown in Fig. 6.6. Both the wing and the horizontal tail of this aircraft have rectangular planforms. The aircraft tail volume ratio $V_H = 1.0$ and $x_{cg} - x_{ac} = 0.01$. The aircraft is flying at an angle of attack such that its main wing generates $C_L = 1.0$. $C_{Ma_c}$ for the cambered wing is $-0.0025$. What must the horizontal tail lift coefficient be, in order to trim the aircraft?

Solution (6.3)
**Dynamic Longitudinal Stability**

**Longitudinal Dynamic Modes**

- The dynamic longitudinal motion of a statically stable aircraft follows a pattern similar to that shown in fig. 6.5a. The graph in fig. 6.5a could easily be a plot of an aircraft's pitch attitude as it encounters a disturbance, such as a wind gust, and then moves back toward its initial condition. However, if we look at how other parameters vary during an aircraft's response to a gust we will typically discover two separate oscillations, largely independent of each other, going on at the same time. These oscillations are called *dynamic modes*.

**Short-Period Mode**

- The first mode that we would typically notice in an aircraft's response to a longitudinal disturbance is called the *short-period mode*. As its name implies, the short-period mode has a relatively short period and therefore a relatively high frequency. The mode involves almost exclusively an angle of attack oscillation, driven by the aircraft's \( C_{M \alpha} \).
- Because the period of this mode is often shorter than the pilot's response time, this type of oscillation can be very difficult for a pilot to correct. In such cases, designers frequently use fly-by-wire flight control systems with artificial damping features to correct the problem.

**Phugoid**

- The second longitudinal dynamic mode is the *phugoid*. This mode involves a flight-path angle/airspeed oscillation with almost no change in angle of attack. The main damping of the phugoid comes from changes in drag resulting from airspeed changes. Therefore high-performance, low-drag aircraft often have very lightly damped phugoids. Gliders often can avoid this by simply launching the glider at its equilibrium speed and flight-path angle.