UNIT D: Applied Aerodynamics

ROAD MAP . . .

- **D-1**: Aerodynamics of 3-D Wings
- **D-2**: Boundary Layer and Viscous Effects
- **D-3**: XFLR (Aerodynamics Analysis Tool)

Unit D-1: List of Subjects

- Infinite v.s. Finite Wings
- Wing Tip Vortices
- Downwash and Induced Drag
- Lift and Downwash
- The Total Drag
- 3-D Drag Polar
- Change in the Lift Curve
- Swept Wings
- Flaps
INFINITE V.S. FINITE WINGS

Infinite wing: the span stretched from $-\infty$ to $+\infty$.

- An airfoil represents a **unit span (width of 1)** of an infinite wing.

Finite wing: 3-D effects (wing tip vortices and spanwise flows) are all included.

- For a finite wing, we need to consider a parameter, called **aspect ratio** of the wing:

$$AR = \frac{b^2}{S}$$
**FINITE WINGS**

The pressure difference between top and bottom surfaces of wing causes a pair of trailing circular motion flows, called the **wing-tip vortices**.

The wing-tip vortices tend to drag the surrounding air around with them, and this secondary movement induces a small velocity component in the downward direction at the wing, called the **downwash**.

The existence of downwash creates an additional drag on wings that cannot be expected on 2-D airfoil (called, **induced drag**).

- If a careful and detailed design/selection of 2-D airfoil is completed, one can still do a poor job designing a 3-D wing . . . resulting in very high induced drag. The quality of wing performance depends on **“how to minimize the induced drag”** by carefully choosing 3-D shape of the wing.

Induced drag depends on:
- Aspect Ratio (AR) of the wing
- How efficient is your wing (called, the **span efficiency factor**), in which depends on your wing taper, wing sweep, wingtip design, twist, dihedral, etc. etc.
- Lift coefficient ($C_L$) of the wing: **IRONICALLY** . . . if your wing produces large amount of lift, it increases the induced drag . . . hence, the induced drag is often called, **“drag due to lift.”**
Downwash and Induced Drag

INDUCED DRAG

The downwash (w) will have several consequences:

- Angle of attack of the airfoil sections of the wing (α_{eff}) is referenced from the local flow direction, which is reduced (by the amount of α_{i}) in comparison to the angle of attack of the wing (α) referenced to V_{∞}.

- There is an increase in the drag, called the **induced drag**. Because of the local relative wind is canted downward, the lift vector itself is “tilted back,” hence it contributes a certain component of force parallel to V_{∞} (drag force).

Induced drag can be obtained from the figure as: 

\[ D_i = L \sin \alpha_i \]

Usually the induced angle of attack (α_{i}) is small, hence for small angles:

\[ \sin \alpha_i \approx \alpha_i, \text{ so } D_i = L \alpha_i \]
LIFT DISTRIBUTION ALONG THE SPAN OF THE WING

If we observe a 3-D wing, the lift is distributed along the span. This is called, the **lift distribution**. The lift distribution depends on:

1. Varying the chord length along the span (tapered wing)
2. Varying the angle of attack along the span (geometric twist)
3. Varying the airfoil along the span of the wing (aerodynamic twist)

Let us first look at an “**ideal**” case. An “elliptical” lift distribution produces a **uniform downwash distribution** along a span of the wing. In such a case:

\[
\alpha_i = \frac{C_L}{\pi AR}
\]

where, \(C_L\): lift coefficient of the finite wing

\(AR\): aspect ratio = \(b^2/S\)

Let us substitute this into the equation of induced drag:

\[
D_i = L\alpha_i = L\frac{C_L}{\pi AR}
\]

Note that: \(L = q_\infty S C_L\), so:

\[
D_i = q_\infty S \frac{C_L^2}{\pi AR}
\]

or

\[
\frac{D_i}{q_\infty S} = \frac{C_L^2}{\pi AR}
\]

Therefore, the induced drag coefficient:

\[
C_{D,i} = \frac{C_L^2}{\pi AR}
\]

- Note: this is the “**ideal case**” (elliptical lift distribution across the wing span)
SPAN EFFICIENCY FACTOR

An “elliptical” lift distribution produces a uniform downwash distribution along a span of the wing (ideal case). An elliptical lift distribution can usually be achieved by having an elliptical wing planform. However, not all airplanes have elliptic planform.

In general, a span efficiency factor ($e$) is defined, such that:

$$ C_{D,i} = \frac{C_L^2}{\pi e AR} $$

- For elliptical planform wings (or elliptical “lift distribution”): $e = 1$
- For all other wing planforms: $e < 1$

In conclusion, the total drag on a finite wing is:

$$ C_D = c_d + \frac{C_L^2}{\pi e AR} $$

The total drag of a finite wing = Profile drag (drag of 2-D airfoil) + Induced drag (3-D effect)

The induced drag is often called the drag due to lift.
THE DRAG POLAR

The induced drag \( (C_{D,i}) \) is the drag due to lift (3-D effect)

The profile drag \( (c_d) \) is a drag on a 2-D airfoil, from which includes: (i) drag due to skin friction (or parasite) drag and (ii) drag due to pressure (or separation) drag:

\[
c_d = c_{d,f} + c_{d,p}
\]

The total drag on an airplane is:

\[
C_D = C_{D,\text{profile}} + C_{D,\text{induced}} = c_d + \frac{C_L^2}{\pi e \text{AR}}
\]

The plot of \( C_L \) vs. \( C_D \) is called the drag polar.

- The drag polar provides important information of “how much drag is associated with a given increase of lift.”
- In aircraft design, usually several varieties of drag polar is presented (clean, flap up or down, gear up or down for TO or L).
Class Example Problem D-1-1

Related Subjects . . . “The Total Drag”

Consider a flying wing (such as Northrop B-2) with a wing area of 200 m$^2$, aspect ratio of 10, span efficiency factor of 0.95, and NACA 4412 airfoil. The weight of the airplane is $7.5 \times 10^5$ N. For the level-flight condition (assume $L = W$ and minimum profile drag) with 0.5 Mach at 10 km altitude, estimate the total drag on the aircraft.

From NACA 4412 airfoil data, the minimum profile drag of the airfoil is:
$c_d = 0.006$ (for $Re = 9.0 \times 10^6$).

At level-flight condition: $L = W = 7.5 \times 10^5$ N

At 10 km altitude:
$\rho_\infty = 0.4135$ kg/m$^3$ and $a_\infty = 299.5$ m/s
$\Rightarrow$ Mach 0.5 = 299.5(0.5) = 149.75 m/s

The dynamic pressure:
$q_\infty = \frac{1}{2} \rho_\infty V_\infty^2 = \frac{1}{2} (0.4135)(149.75)^2 = 4,636.382$ N/m$^2$

The lift coefficient can be calculated as:
$C_L = \frac{L}{q_\infty S} = \frac{7.5 \times 10^5}{(4,636.382)(200)} = 0.8088$
Class Example Problem D-1-1 (cont.)

Related Subjects . . . “The Total Drag”

The total drag coefficient is the sum of minimum profile drag coefficient and the coefficient of drag due to lift (induced drag coefficient).

Therefore:

\[
C_D = c_d + \frac{C_L^2}{\pi eAR} = 0.006 + \frac{(0.8088)^2}{\pi (0.95)(10)} = 0.02792
\]

The total drag is, therefore,

\[
D = q_c S C_D = (4,636.382)(200)(0.02792) = 25,890 \text{ N}
\]
Change in the Lift Curve (1)

Due to downwash, the effective angle of attack of an airfoil \( \alpha_{\text{eff}} \) is less than the geometric angle of attack \( \alpha \), such that: 
\[
\alpha_{\text{eff}} = \alpha - \alpha_i
\]

The induced angle of attack is given by: 
\[
\alpha_i = \frac{C_i}{\pi e AR}
\]

Let us define a lift curve slope of an airfoil \( (a_0) \): 
\[
a_0 = \frac{dC_i}{d\alpha}
\]

Also, a lift curve slope of a finite wing \( (a) \): 
\[
a = \frac{dC_L}{d\alpha}
\]

Let us just use the effective angle of attack \( \alpha_{\text{eff}} \) for the finite wing lift curve slope: 
\[
a_0 \frac{dC_L}{d\alpha_{\text{eff}}} = \frac{dC_L}{d\alpha} (\alpha - \alpha_i)
\]

If we integrate this equation: 
\[
C_L = a_0 (\alpha - \alpha_i) + \text{constant} \quad \text{(eqn. 1)}
\]

\[
a = \frac{a_0}{1 + a_0 / (\pi e AR)}
\]

(for low aspect ratio wing: \( AR < 4 \))

\[
a = \frac{a_0 \cos \Lambda}{\sqrt{1 + \left[ \frac{(a_0 \cos \Lambda)}{(\pi e AR)} \right]^2 + \left[ \frac{(a_0 \cos \Lambda)}{(\pi e AR)} \right]}}
\]

(for swept wing
With \( c/2 \) sweep angle \( \Lambda \))
Change in the Lift Curve (2)

CHANGE IN THE LIFT CURVE (2)

Let us use the definition of induced angle of attack: \( \alpha_i = \frac{C_l}{\pi e AR} \)

Now eqn. 1 becomes: \( C_L = a_0 \left( \alpha - \frac{C_l}{\pi e AR} \right) + \text{constant} \)

Solving this equation for \( C_L \) yields: \( C_L = \frac{a_0 \alpha}{1 + a_0 / \pi e AR} + \frac{\text{constant}}{1 + a_0 / \pi e AR} \)

Differentiating this equation with respect to \( \alpha \): \( \frac{dC_L}{d\alpha} = \frac{a_0}{1 + a_0 / \pi e AR} \)

Recall that \( a = \frac{dC_L}{d\alpha} \), therefore: \( a = \frac{a_0}{1 + a_0 / \pi e AR} \)

CORRECTIONS FOR HIGH ASPECT RATIO AND WING SWEEP

For low aspect ratio wing (\( AR < 4 \)): \( a = \frac{a_0}{\sqrt{1 + \left( a_0 / \pi e AR \right)^2 + a_0 / \pi e AR}} \)

For swept wing (with half chord line sweep angle \( \Lambda \)): \( a = \frac{a_0 \cos \Lambda}{\sqrt{1 + \left[ (a_0 \cos \Lambda) / (\pi e AR) \right]^2 + [(a_0 \cos \Lambda) / (\pi e AR)]}} \)
From NACA 23012 airfoil data, the lift curve slope of this airfoil is:

\[ a_0 = \frac{0.52 - 0.1}{4 - 0} = 0.105 \text{ /deg} = 0.105(180/\pi) = 6.016 \text{ /rad} \]

Let us convert it to the finite wing:

\[ a = \frac{a_0}{1 + \frac{a_0}{\pi eAR}} \]

\[ = \frac{6.016}{1 + \frac{6.016}{\pi (0.95)(10)}} \]

\[ = 5.007 \text{ /rad} = 5.007(\pi/180) = 0.087 \text{ /deg} \]
Using the lift curve slope of the finite wing:

\[ C_L = a(\alpha - \alpha_{\alpha=0}) = 0.087[4 - (-1.7)] = 0.498 \]

At this lift coefficient, corresponding profile drag is \( c_d = 0.006 \).

Therefore,

\[ C_d = c_d + \frac{C_L^2}{\pi e AR} = 0.006 + \frac{(0.498)^2}{\pi(0.95)(10)} = 0.014 \]}
Swept Wings (1)

**SWEPT WINGS**

By sweeping the wings of subsonic aircraft, \textbf{drag divergence is delayed} to higher Mach numbers.

\[ M_{\text{cr}} \text{ for airfoil} < \frac{M_{\text{cr}} \text{ for swept wing}}{\cos \Lambda} < M_{\text{cr}} \text{ for airfoil} \]

(1) By sweeping the wing, each airfoil section of the wing “sees” unparallel freestream velocity. As a result, each airfoil section needs to deal only with a velocity component normal to its direction (which is less than freestream itself).

(2) By sweeping the wing, the freestream “sees” an increased geometric chord length. Hence, the effective thickness ratio is reduced.

(3) By sweeping the wing, \textbf{the lift curve slope is effectively reduced}. 

\[ \theta = 45^\circ \]

\[ \theta = 30^\circ \]
DRAG COUNT

A drag count is a single unit of drag as defined by aerospace engineers. A drag count is 1/10,000 of a $C_D$. For example, if a drag is increased by 0.01, it is called, “100 count of drag increase.” Drag count is used as a **crude measure for the change in drag coefficient** (it is not a direct measure of drag as it is not associated with any reference area).

EFFECTS OF WING SWEEP AND THICKNESS RATIO

By decreasing the thickness ratio from 9% => 6% => 4%:

- $t/c$ 9% => 6%: approximately 100 drag count decrease
- $t/c$ 6% => 4%: approximately 70 drag count decrease

By increasing wing sweep from 11° => 35° => 47°:

- $\Lambda$ 11° => 35°: approximately 30 drag count decrease
- $\Lambda$ 35° => 47°: approximately 50 drag count decrease
AIRCRAFT STALL SPEED

The minimum sustainable airspeed for an aircraft without stall is called the stall speed. This is an important parameter, as this is a driving factor of TO-L performance (also a flight safety).

From the definition of lift:

\[ L = q_\infty S C_L = \frac{1}{2} \rho_\infty V_\infty^2 S C_L \Rightarrow V_\infty = \sqrt{\frac{2L}{\rho_\infty S C_L}} \]

In steady, level flight, the lift is just sufficient to support the weight \( W \) of the aircraft.

That is, \( L = W \). Therefore:

\[ V_\infty = \sqrt{\frac{2W}{\rho_\infty S C_L}} \]

In order to minimize this flight airspeed (minimum flight speed is usually equal to the stall speed), one needs to maximize the lift coefficient.

Thus,

\[ V_{\text{stall}} = \sqrt{\frac{2W}{\rho_\infty S C_{L, \text{max}}}} \]
HIGH LIFT DEVICES

Maximum lift coefficient of an airfoil ($c_{L, \text{max}}$) is in the range of 1.4 – 1.5. However, this is 2-D airfoil.

3-D finite wing’s maximum lift coefficient is slightly less than that. Usually, airplanes need $C_{L, \text{max}}$ in the range of 1.8 – 2.4 for TO-L operations. It is required to deploy high lift devices to achieve this high $C_{L, \text{max}}$

- Trailing edge devices are called “flaps”
- Leading edge devices are called “slats”

By deploying the flaps/slats, one can increase $C_{L, \text{max}}$.

However, the stall angle of attack is also decreased (airplane stalls in much lower angle of attack).
VARIATION OF HIGH LIFT DEVICES

① Airfoil only
② Plain Flap
③ Split Flap
④ Leading Edge Slat
⑤ Single-Slotted Flap
⑥ Double-Slotted Flap
⑦ Double-Slotted Flap + Leading Edge Slat
⑧ Double-Slotted Flap + Leading Edge Slat + Top Surface Boundary layer Suction
Class Example Problem D-1-3

Related Subjects . . . “Swept Wings” and “Flaps”

For swept wings and flaps, what are positive (favorable) and negative (adverse) aerodynamic effects of each device.

Pros for swept wings:
- Delay critical Mach number to a much higher Mach number.
- Decrease drag at transonic/supersonic flights.

Cons for swept wings:
- Lift curve slope is decreased – for maintaining the same stall speed, the wing area needs to be increased.
- For integrated wing-tank configuration, the shift of CG is an issue.

Pros for flaps:
- Increase maximum lift coefficient (thus, decrease stall speed).

Cons for flaps:
- Decrease stall angle of attack.
- Safe TO-L operation entirely depends upon flaps. The malfunction of flaps usually links directly to the cause of serious problems.
UNIT D: Applied Aerodynamics

ROAD MAP . . .

D-1: Aerodynamics of 3-D Wings
D-2: Boundary Layer and Viscous Effects
D-3: XFLR (Aerodynamics Analysis Tool)

Unit D-2: List of Subjects

- What is Boundary Layer?
- Laminar v.s. Turbulent Flow
- Flat Plate Laminar Boundary Layer
- Boundary Layer Transition to Turbulent Flow
- Ideal v.s. Real Flow
- Flow Separation
- Attached v.s. Separated Flow
- Flow Separation and Stall
- Viscous Effects on Drag
BOUNDARY LAYER

Boundary layer is a thin layer formed on the surface of a solid body in a flow field. Due to the viscosity, the velocity within the boundary layer changes with respect to the vertical distance from the surface (usually non-linearly).

The boundary layer “grows” as the flow moves over the body (the boundary layer thickness is defined as: $\delta$)

Within the boundary layer, the velocity varies from zero (surface) to the free stream (edge of the layer): this is called, the “boundary layer velocity profile”

At the surface (wall) of the body, wall shear stress can be given as: $\tau_w = \mu \left( \frac{dV}{dy} \right)_{y=0}$

This wall shear stress is one of the major contributor to the vehicle drag, called the “skin friction drag”
LAMINAR V.S. TURBULENT FLOWS

**Laminar flow:** where the streamlines are smooth and regular, and a fluid element moves smoothly along a streamline

**Turbulent flow:** where the streamlines break up and a fluid element moves in a random, irregular, and chaotic fashion (means streamlines cannot be defined as it is unsteady in nature)

**Transition** from laminar to turbulent occurs as Reynolds number of the flow increases (usually the boundary layer “transition point” is defined, based on the **transition Reynolds number:** \( \text{Re}_{tr} \)
LAMINAR BOUNDARY LAYER

The solution known as "Blasius" solution of flat plate laminar boundary layer (flat plate means: zero pressure gradient)

\[ C_f = \frac{1.328}{\sqrt{Re}} \text{ laminar} \]

\[ \delta = \frac{5.2x}{\sqrt{Re_x}} \text{ laminar} \]

\[ c_{f,s} \equiv \frac{\tau_w}{0.5 \rho_x V_{x,\infty}^2} \equiv \frac{\tau_w}{q_{\infty}} = 0.664 \] (laminar flat plate boundary layer)

(Lower case \( c_{f,s} \) represents the skin friction drag coefficient per unit length at a location of \( x \))

The skin friction coefficient (due to the wall shear stress) will cause the total skin friction drag for the flat plate of length \( L \):

\[ D_f = \int_0^L \tau_w dx = 0.664 q_{\infty} \int_0^L \frac{dx}{\sqrt{Re_x}} = \frac{0.664 q_{\infty}}{\sqrt{\rho_x V_{x,\infty}/\mu_x}} \int_0^L \frac{dx}{\sqrt{\rho_x V_{x,\infty}/\mu_x}} = 1.328 q_{\infty} \sqrt{L} \]

Total skin friction drag coefficient of the flat plate of length \( L \) can be defined as:

\[ C_f \equiv \frac{D_f}{q_{\infty} S} = \frac{D_f}{q_{\infty} L(1)} = \frac{1.328 q_{\infty} \sqrt{L}}{q_{\infty} L \sqrt{\rho_x V_{x,\infty}/\mu_x}} \frac{1.328}{\sqrt{\rho_x V_{x,\infty} L/\mu_x}} = \frac{1.328}{\sqrt{Re_L}} \]

(Upper case \( C_f \) represents the total skin friction drag coefficient of the given flat plate of length \( L \))
Class Example Problem D-2-1

Related Subjects . . . “Laminar Boundary Layer”

Consider a flow of air over a flat plate (5 cm long in the flow direction, and 1 m wide). The freestream conditions correspond to standard sea-level, and the flow velocity is 120 m/s. Assuming the laminar flow, determine the followings:

(a) The boundary layer thickness at the trailing edge of the plate

(b) The drag force (due to skin friction) developed on the plate

(a) At the trailing edge \((x = 5 \text{ cm})\) of the plate, the Reynolds number is:

\[
\text{Re}_x = \frac{\rho_\infty V_\infty x}{\mu_\infty} = \frac{(1.225 \text{ kg/m}^3)(120 \text{ m/s})(0.05 \text{ m})}{1.789 \times 10^{-5} \text{ kg/(m \cdot s)}} = 4.11 \times 10^5
\]

Hence, the boundary layer thickness is:

\[
\delta = \frac{5.2 x}{\sqrt{\text{Re}_x}} = \frac{5.2 (0.05 \text{ m})}{\sqrt{4.11 \times 10^5}} = 4.06 \times 10^{-4} \text{ m} = 0.0406 \text{ cm}
\]

(b) The total skin friction drag coefficient is:

\[
C_f = \frac{1.328}{\sqrt{\text{Re}_x}} = \frac{1.328}{\sqrt{4.11 \times 10^5}} = 2.07 \times 10^{-3}
\]

Note that:

\[
q_\infty = 0.5 \rho_\infty V_\infty^2 = 0.5 \left(1.225 \text{ kg/m}^3\right)(120 \text{ m/s})^2 = 8,820 \text{ N/m}^2
\]

\[
S = 0.05 \text{ m} = 0.05 \text{ m}^2
\]

Therefore, the skin friction drag force can be obtained by:

\[
D_f = q_\infty S C_f = 8,820 \text{ N/m}^2 \left(0.05 \text{ m}^2\right) \left(2.07 \times 10^{-3}\right) = 0.913 \text{ N}
\]

Note that this is the skin friction drag force on a single surface. Since the flat plate have two surfaces (top & bottom surfaces), the total skin friction drag force is:

\[
D_f \text{ (total)} = 2(0.913 \text{ N}) = 1.826 \text{ N}
\]
TURBULENT BOUNDARY LAYER

Turbulent boundary layer is thicker than the laminar boundary layer, under the same condition of flow

Turbulent boundary layer is often very difficult to analyze: heavily dependent on experimental results

TRANSITION FROM LAMINAR TO TURBULENT BOUNDARY LAYER

The adverse pressure gradient, rough surface (skin), and many other factors determine boundary layer transition

The physical mechanism of boundary layer transition, even for a simple flat plate, is still not completely understood. The boundary layer transition mechanism is still the area of intensive research in aerodynamics today
Consider a flow of air over a flat plate (5 cm long in the flow direction, and 1 m wide). The freestream conditions correspond to standard sea-level, and the flow velocity is 120 m/s. If the flow is turbulent, determine the followings:
(a) The boundary layer thickness at the trailing edge of the plate
(b) The drag force (due to skin friction) developed on the plate

(a) At the trailing edge ($x = 5$ cm) of the plate, the Reynolds number is:

$$
Re_x = \frac{\rho V x}{\mu} = 4.11 \times 10^5
$$

Hence, the boundary layer thickness is:

$$
\delta = \frac{0.37x}{Re_x^{0.2}} = \frac{0.37(0.05 \text{ m})}{(4.11 \times 10^5)^{0.2}} = 0.37 \times 10^{-3} \text{ m} = (0.139 \text{ cm})
$$

Note that the turbulent boundary layer is **3.42 times thicker** than laminar flow.

(b) The total skin friction drag coefficient is:

$$
C_f = \frac{0.074}{Re_{L}^{0.2}} = \frac{0.074}{(4.11 \times 10^5)^{0.2}} = 0.00558
$$

Note that:

$$
q_{\infty} = 0.5 \rho_{\infty} V_{\infty}^2 = 0.5 \left(1.225 \text{ kg/m}^3\right) \left(120 \text{ m/s}\right)^2 = 8,820 \text{ N/m}^2
$$

$$
S = 0.05 \text{ m} (1) = 0.05 \text{ m}^2
$$

The skin friction drag force can be obtained by:

$$
D_f = q_{\infty} SC_f = 8,820 \text{ N/m}^2 \left(0.05 \text{ m}^2\right) (0.00558) = 2.46 \text{ N}
$$

Note that this is the skin friction drag force on a single surface. Since the flat plate have two surfaces (top & bottom surfaces), the total skin friction drag force is:

$$
D_f \text{ (total)} = 2 (2.46 \text{ N}) = 4.92 \text{ N}
$$

Note that the skin friction drag force of the turbulent boundary layer is **2.7 times larger** than laminar flow.
The general solution procedure:

(1) Calculate $D_f$ for the combined area $A + B$, assuming that the flow is completely turbulent

(2) Obtain the turbulent $D_f$ for the area $B$ only, by calculating the turbulent $D_f$ for area $A$ and subtracting this from the result of part (1)

(3) Calculate the laminar $D_f$ for the area $A$

(4) Add results from parts (2) & (3) to obtain total drag on the complete surface area $A + B$

(5) Note that the Wright Flyer I is a biplane and has total 4 surfaces (top & bottom and 2 wings), so the total skin friction drag on the complete biplane wing configuration will be the $\times 4$ of the skin friction drag on a single surface, calculated from part (4)
**Ideal v.s. Real Flow**

**RECALL: [IDEAL V.S. REAL FLOW: CIRCULAR CYLINDER] (UNIT B-3)**

As we learned in Unit B-3, the pressure distribution around circular cylinder (ideal flow) is symmetric, thus no lift and no drag will be produced. This is known as d’ Alembert’s paradox.

Flow separation, due to the adverse pressure gradient, and resulting complex wake region induces the non-symmetric pressure distribution, thus “pressure drag due to separation” is the main drag contributor of real flows around circular cylinder.

**IDEAL V.S. REAL FLOW: AERODYNAMICS OF AIRFOILS AND WINGS**

A very similar analogy (that we employed for the analysis of flow over a circular cylinder) can also be applied for the aerodynamics of airfoils and wings. The formation of boundary layer and its behavior (transition from laminar to turbulent flows, as well as the flow separation, due to the pressure gradient) plays very important roles in generations of aerodynamic lift and drag of airfoils and wings.
**Flow Separation**

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**RECALL: [PRESSURE GRADIENT] (UNIT B-1)**

$p$ (pressure) is a function of spatial coordinates: $p = p(x, y)$

The mathematical representation: **pressure gradient**, $\nabla p$, is a vector such that:

- Magnitude = maximum rate of change of $p$ per unit length of the coordinate space at the given point
- Direction = direction of the maximum rate of change of $p$ at the given point

Using pressure gradient, directional pressure change can be given: $\frac{dp}{ds} = \nabla p \cdot \hat{n}$

(This is “how much change of pressure takes place in the direction specified by the line vector $\hat{n}$”: more commonly called physical “pressure gradient”)

**FLOW SEPARATION**

Pressure gradient: $dp/ds$ (along the surface of an airfoil) may be favorable (negative) or adverse (positive), and adverse pressure gradient causes flow to separate from the surface.
FLOW SEPARATION ON AN AIRFOIL

Flow separation on an airfoil causes:

- A drastic loss of lift (stall)
- A major increase in drag, caused by pressure drag due to separation

FLOW SEPARATION AND STALL

At extremely high angles of attack, the flow separates from the top surface of an airfoil, leading it to stall. Desired “post-stall” characteristics (smooth and gradual stall or sudden violent stall) and ease of recovery are important design factors of airfoil.
RECALL: [LEADING EDGE STALL] (UNIT C-4)

Example: NACA 4412

- Characteristics of relatively thin airfoils with thickness ratios between 10 and 16 percent of the chord length.
- Post-stall: **rapid loss of the lift (not desirable)**

RECALL: [EFFECTS OF AIRFOIL THICKNESS ON STALL] (UNIT C-4)

Thin airfoil: thin airfoil is desired for high-speed applications (from high transonic to supersonic), in order to minimize profile and wave drags.

Thick airfoil (example: NACA 4421): thick airfoil is usually desired, especially for low-speed applications (low subsonic), due to the favorable stall characteristics (**Trailing Edge Stall**).
Viscous Effects on Drag

The presence of viscosity in a flow produces two sources of drag:

- **Skin friction drag** (due to shear stress on the surface of the body, called the wall shear stress) => this is often called “Parasite Drag”
- **Pressure drag** (due to flow separation) => this is often called “Drag due to Separation”

Trade-off between these two drag components may be possible by carefully engineered design: for example,

- Natural Laminar Flow (NLF) airfoil (NACA 6-series)
- Golf ball dimples

**RECALL: [THE DRAG POLAR] (UNIT D-1)**

The induced drag \( C_{D,i} \) is the drag due to lift (3-D effect)

The profile drag \( C_d \) is a drag on a 2-D airfoil, from which includes: (i) **drag due to skin friction** (or parasite) drag and (ii) **drag due to pressure** (or separation) drag:

\[
c_d = c_{d,f} + c_{d,p}
\]

The total drag on an airplane is:

\[
C_D = "Profile" \text{ Drag} + "Induced" \text{ Drag} = c_d + \frac{C_L^2}{\pi e \ AR}
\]
AE301 Aerodynamics I

UNIT D: Applied Aerodynamics

ROAD MAP . . .

D-1: Aerodynamics of 3-D Wings
D-2: Boundary Layer and Viscous Effects
D-3: XFLR (Aerodynamics Analysis Tool)

This is a tutorial (self-study) material for the COURSE PROJECT
- Not covered in class as a standard lecture of AE301
- Visit XFLR5 Project Homepage (http://www.xflr5.tech/xflr5.htm)

Unit D-3: List of Subjects

- Description of XFLR5
- 2-D (Airfoil Analysis) Examples
- 3-D (Wing Analysis) Examples
Description of XFLR5

• XFLR5 is an analysis tool for airfoils, wings and planes. It includes:
  — Xfoil's Direct and Inverse analysis capabilities
  — Wing design and analysis capabilities based on the Lifting Line Theory, on the Vortex Lattice Method, and on a 3D Panel Method
• The airfoil analysis portion is based on the program XFOIL developed by Professor Mark Drela from MIT. XFLR5 has an updated GUI, so the operation of it is somewhat different than that of XFOIL.
• The XFLR5 program and Guidelines can be downloaded from the project web site:
  • http://sourceforge.net/projects/xflr5/
  • http://www.xflr5.com/xflr5.htm (links to related material)
• More information on XFOIL is available at http://web.mit.edu/drela/Public/web/xfoil/ and a simple tutorial can be found on the course website in the file XFOIL_tutorial.pdf.

This is the material adapted from the Boeing AerosPACE* program
(Aerospace Partners for Advancement in Collaborative Engineering)

XFLR5 Examples

1. NACA 2415
2. NACA 66(2)215 laminar flow airfoil
3. Rectangular Wing
   a) Lifting Line Theory
   b) 3D Panel method
Example 1: NACA 4 digit Airfoil Analysis

- NACA 2412
- Reynolds number 1 million to 10 million in steps of 1 million
- Angle of attack -5 to 10 degrees in steps of .2 degrees
• Start XFLR5
• Click <File> <Direct Foil Design>
• Click <Foil Design> <Naca Foils>
• Enter the 4- or 5-digit name of the airfoil and the number of panels to use

• Click <File> <XFOIL <Direct Analysis>
• Click <Analysis> <BatchAnalysis>.

• Choose Type 1.
• Enter Reynolds number, Mach number and transition information.
• Enter angles of attack
• Click the <Analyze> button.

The program runs through an iterative procedure to solve the problem at each angle of attack. Click close when finished.
1. Click the pressure distribution icon
2. Click analyze
3. Change Angle of Attack

Pressure Distribution

2. Click analyze

Analysis Results
• The program may not converge for a given angle of attack if the solution is particularly complex or if the change from the initial guess or the previous solution is too large. You can increase the maximum number of iterations in the <Analysis><XFOIL Advanced Settings> menu. If there are a small number of angles of attack for which the solution did not converge, that is OK. Just realize that the results for those angles of attack are more unreliable.

• The airfoil is shown in the bottom part of the window at the angle of attack at the end of the sequence you chose. In the upper part of the window is shown the pressure coefficient distribution. To see Cp using an inviscid analysis (panel method), choose <Operating Points><Cp Graph><Show Inviscid Curve> (or <right click> on the Cp graph instead of <Operating Points>). The inviscid Cp distribution shows up as a dashed line, while the solid line shows Cp accounting for viscous boundary layer effects. You can choose a particular angle of attack by clicking on the button on the far right side of the tool bar.

• Check the box in the XDirect pane for Show Pressure to see the local pressure distribution on the airfoil shown as force arrows. Check the Show Boundary Layer box to see the boundary layer thickness on the airfoil surface. Check the Animate box to see a sweep through the angles of attack and to watch the results change.

• Click <Polars><Polar Graphs><All Polar Graphs> to see the five polar plots. The menu that the choice <All Polar Graphs> is on shows what is plotted as figures (1) through (5). (Note: There is a short cut button in the tool bar at the top to switch between polars and the Cp plot.)

• You can use the mouse to zoom in and out and to translate any of the plots.

• To save a plot choose <Right Click><Save View to Image File>. Possible file formats are bitmap, jpeg and png.

• To plot other variables computed by XFOIL, on the Cp plot choose <Right Click><Cp Graph><Current XFOIL Results> and then the name of the variable you want to plot, e.g. <Skin Friction Coefficient>. (The variables D* and Theta refer to d* = d_1 = boundary layer displacement thickness and q = d_2 = boundary layer momentum thickness.)

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**Example 2: NACA 66(2)-215 Airfoil Analysis**

- NACA 66(2)-215 laminar airfoil
- Reynolds number 1 million to 10 million in steps of 1 million
- Angle of attack -5 to 10 degrees in steps of .2 degrees
Airfoil Coordinates for XFLR5

- XFLR5 reads coordinates from a *.dat file
- The points must be in (x,y) pairs, starting at the trailing edge (TE), going to the leading edge (LE), and back to the TE. The points may go over the upper surface and back along the lower surface, or vice versa (the code can figure that out).
- The first line is the airfoil name
- A good source of airfoil data is:
  - http://www.ae.illinois.edu/m-selig/ads/coord_database.html
- Note that some of this data is in the wrong format and must be reordered

Example file naca662215.dat

NACA 66(2)-215
1.000000 0.000000
0.993359 0.001014
0.982368 0.002802
0.969897 0.004996
0.955711 0.007707
0.939801 0.011019
0.922598 0.014866
0.904739 0.019047
0.886614 0.023373
0.868296 0.027778
0.849849 0.032225

........................................
........................................
0.909485 -0.008053
0.926545 -0.005413
0.942539 -0.003293
0.957211 -0.001775
0.970490 -0.000802
0.982471 -0.000252
0.993316 -0.000019
1.000000 0.000000

Airfoil Analysis with Imported Coordinate Data File (.dat)

- Click <File> < new project>
- Click <File> < open> naca662215.dat
- Click <File> < direct foil design> to see airfoil
- Click <File> < Xfoil direct analysis>
- Click <Analysis> <BatchAnalysis>.
- Choose Type 1.
- Enter Reynolds number, Mach number and transition information.
- Enter angles of attack
  - Click the <Analyze> button.

Same as Example #1
Example 3 (a): NACA 2415
Rectangular Wing
Lifting Line Theory

Christen A-1 Husky
Wing Analysis

- Click <File> < new project>
- Run Example #1 NACA 2415 airfoil
- Click <file> <wing and plane Design
- Click <wing and plane> <new wing design>
- The wing edition window pops up

- Click Symmetry and right side
- Add dimensions for the right half wing
- Click <foil> and choose NACA2415
- Add the panel numbers and click the distribution and choose cosine
- Check calculated quantities on right side
- Click ,save and close.
- Click <polars><define analysis
- In pop-up window
  - Choose auto name, Type 1
  - Input Free stream speed
    - Check calculated Re numbers to be sure they are in the range of the airfoil analysis. You will get an error if they are out of range.
  - Choose international units
  - Choose LLT – lifting Line Theory
- Click OK

- On Pop-up
- Set angle of attack range
- Click Analyze
Polar Plots

Spanwise Properties
Right Click on graph, choose current op point, export
Generates file of data

Example #3
V = 10.0 m/s

XFLR5 v6.06

Example #3
TI-10.0 m/s-LLT
Q infl = 10.000000 m/s
Alpha = 4.000000

X = 0.000000

Bend = 1182.365112

XCP = 0.580060     YCP = 0.000000

XNP = 0.000000

Bend = 1182.365112

Example #3

x-span Chord Ai Cl PCd ICd CmGeom CmAirf XTrtop XTrBot XCP BM
-6.8150 1.7250 -4.594 0.172674 0.006612 0.013845 -0.094896 -0.051747 0.6109 0.4782 0.5500 0.0000
-6.5623 1.7250 -3.350 0.308041 0.006376 0.018009 -0.127697 -0.050674 0.5468 0.6522 0.4133 0.7827
-6.1479 1.7250 -2.458 0.404226 0.006369 0.017345 -0.150668 -0.049605 0.5049 0.7636 0.3682 6.5532
-6.5822 1.7250 -1.848 0.469725 0.006398 0.016086 -0.166086 -0.048677 0.4806 0.8337 0.3484 26.2776
-4.8790 1.7250 -1.438 0.513307 0.006472 0.012887 -0.176195 -0.047924 0.4645 0.8757 0.3377 72.9358
-4.0557 1.7250 -1.167 0.541930 0.006549 0.010334 -0.182373 -0.047339 0.4537 0.9141 0.3314 161.6461
-3.1325 1.7250 -0.989 0.560369 0.006636 0.009673 -0.189913 -0.046927 0.4451 0.9141 0.3276 307.2760
-2.322 1.7250 -0.882 0.571906 0.006668 0.008799 -0.195999 -0.046744 0.4414 0.9222 0.3255 522.0347
-1.0794 1.7250 -0.823 0.578211 0.006685 0.008303 -0.191067 -0.046444 0.4394 0.9266 0.3244 813.3718
-0.0000 1.7250 -0.804 0.580211 0.006690 0.008143 -0.195332 -0.046112 0.4388 0.9280 0.3240 1182.3651
1.0794 1.7250 -0.823 0.578211 0.006685 0.008303 -0.191067 -0.046444 0.4394 0.9266 0.3244 813.3718
2.1322 1.7250 -0.882 0.571906 0.006668 0.008799 -0.189999 -0.046744 0.4414 0.9222 0.3255 522.0347
3.1325 1.7250 -0.989 0.560369 0.006636 0.009673 -0.186913 -0.046927 0.4451 0.9141 0.3276 307.2766
4.0557 1.7250 -1.167 0.541930 0.006549 0.010334 -0.182373 -0.047339 0.4537 0.9141 0.3314 161.6461
4.8790 1.7250 -1.438 0.513307 0.006472 0.012887 -0.176195 -0.047924 0.4645 0.8757 0.3377 72.9358
5.5822 1.7250 -1.848 0.469725 0.006398 0.016086 -0.166086 -0.048677 0.4806 0.8337 0.3484 26.2776
6.1479 1.7250 -2.458 0.404226 0.006369 0.017345 -0.150668 -0.049605 0.5049 0.7636 0.3682 6.5532
6.5623 1.7250 -3.350 0.308041 0.006376 0.018009 -0.127697 -0.050674 0.5468 0.6522 0.4133 0.7827
6.8150 1.7250 -4.594 0.172674 0.006612 0.013845 -0.094896 -0.051747 0.6109 0.4782 0.5500 0.0000
Example 3(b): NACA 2415
Rectangular Wing
3D Panel Method

• Continuing from the LLT analysis
• Click <polars><define an analysis
• Choose 3D Panels
• Click OK
• Set angle attack range
• Click Analyze
Right Click on graph, choose current op point, export
Generates file of data with pressures on all panels