UNIT A: Aerodynamics and Airfoils

ROAD MAP . . .

A-1: Fundamentals Review
A-2: Airfoil Characteristics

Unit A-1: List of Subjects

- Continuity Equation
- Euler’s & Bernoulli’s Equations
- Airspeed Indicators
- ICeT/ICeTG
- Low-Speed Wind Tunnels
- Airfoils
- Boundary Layer & Skin Friction Drag
- Flow Transition & Separation

Brandt, et.al., Introduction to Aeronautics: A Design Perspective
Chapter 3: Aerodynamics and Airfoils
3.1 Design Motivation
3.2 Basic Aerodynamics
3.3 Airspeed Indicators
3.4 Viscous Flow
Streamlines & Stream Tube

- Each streamline in a flow field is drawn so that every point along its length the local velocity vector is tangent to it. A tube composed of streamlines is called a stream tube. In a steady flow, each streamline will also be the path taken by a particle of air (path line) or a series of particles follows one after another (streak line).
- A steady flow is defined as one in which the flow properties at each point in the flow field do not depend on time.
- The point where the flow stops (zero velocity) is called a stagnation point. The streamline leading to the stagnation point is called a stagnation streamline.
- If, at each point along a streamline, there is no variation in the properties in a plane perpendicular to the flow direction, the flow is considered as one dimensional.

Continuity Equation

- Considering a flow in a stream tube, the rate at which mass is flowing through a plane perpendicular to a steady one-dimensional flow is given by:
  \[
  \dot{m} = \rho A \dot{V} \quad (3.1) \quad \text{Mass Flow Rate}
  \]
  \[
  \rho : \text{fluid density}
  \]
  \[
  A : \text{cross-sectional area}
  \]
- Between station 1 & 2 of the given stream tube:
  \[
  \rho_1 A_1 \dot{V}_1 = \rho_2 A_2 \dot{V}_2 \quad (3.2) \quad \text{Continuity Equation of a steady one dimensional flow field}
  \]
- Due to the change of cross-sectional area, the velocity change occurs between station 1 & 2 of the given stream tube.
Example A-1-1
(Continuity Equation)

Example 3.1

Air flows through a tube that changes cross-sectional area in a way that is similar to the tube illustrated in Fig. 3.3. At a point in the tube (station 1) where the cross-sectional area is 1 m$^2$, the air density is 1.2 kg/m$^3$, and the flow velocity (magnitude) is 120 m/s. At another point in the tube (station 2) the cross-sectional area is 0.5 m$^2$, and the air density has decreased to 1.0 kg/m$^3$. What is the mass flow rate through the tube, and what is the flow velocity (magnitude) at station 2?

Solution (3.1):
Euler’s & Bernoulli’s Equations

Euler’s Equation

• Considering a **steady, inviscid** flow without the effect of any body forces (i.e., **zero gravity**), a small cubic element (a fluid particle) within a flow field is under the Newton’s second law:

\[
\sum F = m \frac{dV}{dt} = pdA - (p + dP)dA
\]

with the volume of the particle of \( m = \rho dsdA \), as well as, \( \frac{dV}{dt} = \frac{dV}{ds} \frac{ds}{dt} = V \frac{dV}{ds} \) then:

\[
\rho dA - (p + dP)dA = \rho dAdV \frac{dV}{ds}
\]

\[
\Rightarrow -dP = \rho V dV
\]  
(3.3) **Euler’s Equation**

Bernoulli’s Equation

• For an **incompressible** flow, eq(3.3) can be integrated between two arbitrary points (station 1 & 2) along a streamline:

\[
\int_{1}^{2} dP = -\rho \int_{1}^{2} V dV \Rightarrow P_2 - P_1 = -\rho \left[ \frac{V^2}{2} \right]_{1}
\]

\[
\Rightarrow \frac{P_1 + \rho V_1^2}{2} = P_2 + \frac{\rho V_2^2}{2} = P_0
\]  
(3.4) **Bernoulli’s Equation**

\[
q \equiv \frac{\rho V^2}{2} : \text{dynamic pressure}
\]

• Each component of eq(3.4) can be interpreted as different type of pressure in a flow field:

\[
P + \frac{\rho V^2}{2} = P + q = P_0 \Rightarrow "\text{STATIC PRESSURE"} + "\text{DYNAMIC PRESSURE"} = "\text{TOTAL PRESSURE"}
\]

Summary: Assumptions (i.e., Limitations) of Euler's & Bernoulli’s Equations

• Euler's equation is based on Newton's second law with assumptions of **steady & inviscid** flow with **no body force**. The application of this equation is limited only along a given **streamline**.

• Bernoulli's equation is based on Euler's equation with assumption of **incompressible** flow. In aerodynamics, the flow field **Mach number of less than 0.3** is considered as **incompressible subsonic** flow.
Airspeed Indicators

Pitot-Static Tube

• A simple airspeed indicating system can be constructed by: Pitot-Static Tube + Manometer with application of Bernoulli's equation.

\[ P_0 = P_\infty + \frac{\rho V^2}{2} \]  

\[ \Rightarrow V_\infty = \sqrt{\frac{2(P_0 - P_\infty)}{\rho}} \]  

\((3.7) \text{ Airspeed Equation } (M < 0.3)\)

Airspeed Indicator

• For aircraft application, a differential pressure gauge is used (instead of manometer). The static and total pressure lines are connected to opposite sides of a diaphragm. The pressure difference causes the diaphragm to deflect. A linkage connected to the diaphragm moves a needle on the gauge dial.

• By calibrating the dial scale in terms of velocity (instead of pressure), the differential pressure gauge becomes an \textit{airspeed indicator}.

Indicated Airspeed \( (V_i) \)

• The airspeed that the needle on the airspeed indicator points at for a given set of flight condition is called (simply) the \textit{indicated airspeed} \( (V_i) \).

• However, usually, this indicated airspeed is \textit{not} the speed at which the aircraft is moving through the air.

• You must understand that the accurate measurement of airspeed requires a series of error corrections and calibrations. The process can be understood as "\textit{Indicated to Calibrated to Equivalent to True}" (ICeT). Often also added "to Ground" (ICeTG) airspeed. The lower case "e" being used as a reminder that equivalent airspeed is usually less than the other airspeeds.
Example A-1-2
(Airspeed Indicators)

Example 3.2
A manometer connected to a pitot-static tube, as in Fig. 3.6, has a difference in the height of the two columns of water of 10 cm when the pitot-static tube is placed in a flow of air at standard sea-level conditions. What is the velocity (magnitude) of the airflow?

Solution (3.2):
ICeT/ICeTG

- First, errors in total pressure measurements for certain conditions (also the indicator itself) will need to be calibrated from indicated airspeed:
\[ V_c = V_i + \Delta V_p \] (3.8) \textbf{Calibrated Airspeed} \( (V_c) \)
\[ \Delta V_p \] : position error or installation error

- Next, the pilot must multiply calibrated airspeed by the "f-factor" (called f-factor correction):
\[ V_e = fV_c \] (3.9) \textbf{Equivalent Airspeed} \( (V_e) \)
\( f \) : compressibility correction "f factor"

- The final correction is multiplying by the square root of the air density ratio:
\[ V_\infty = V_e \sqrt{\frac{\rho_{SL}}{\rho_\infty}} \] (3.10) \textbf{True Airspeed} \( (V_\infty) \)
\( \rho_{SL} \) : air density at sea-level
\( \rho_\infty \) : air density at a given set of flight condition

- Because the density ratio \( \left( \frac{\rho_{SL}}{\rho_\infty} \right) \) is usually less than 1, true airspeed is usually higher than equivalent airspeed \( (V_\infty \geq V_e) \). When at sea-level (on a standard day), \( \rho_{SL}/\rho_\infty = 1 \) and \( V_\infty = V_e \)

- The dynamic pressure is given by:
\[ q = \frac{\rho_\infty V_\infty^2}{2} = \frac{\rho_\infty}{2} \left( V_\infty \sqrt{\frac{\rho_{SL}}{\rho_\infty}} \right)^2 = \frac{\rho_{SL} V_e^2}{2} \] (3.11)

- Hence, equivalent airspeed can alternatively be defined as the airspeed that would produce the same dynamic pressure at sea-level as is measured for the given flight conditions.

- Because \( V_e \) is a "direct measure" of dynamic pressure, it is a very useful standard analytical tool of an aircraft's force-generating capabilities.

- True airspeed is the aircraft's true velocity vector relative to the air mass. Often, the air mass itself can be moving relative to the ground (wind velocity: \( V_{\text{wind}} \)). The aircraft's velocity vector relative to the Earth's surface must be determined in such a case:
\[ V_g = V_\infty + V_{\text{wind}} \] (3.12) \textbf{Ground Speed of Aircraft} \( (V_g) \)
Example A-1-2
(Airspeed Indicators)

Example 3.2
A manometer connected to a pitot-static tube, as in Fig. 3.6, has a difference in the height of the two columns of water of 10 cm when the pitot-static tube is placed in a flow of air at standard sea-level conditions. What is the velocity (magnitude) of the airflow?

(NOTE) 300-kn & 50-kn: (knots, often also in "kts") is a "nautical miles per hour." A nautical mile is based on the circumference of the earth, and is equal to one minute of latitude. It is, in fact, slightly more than a "statute" mile (land measured: commonly used "mile"). The unit conversion is: 1 nautical mile = 1.1508 statute miles. Nautical miles are commonly used for charting and navigating in aeronautics/aviation.

Solution (3.3)

Solution (3.4)
Low-Speed Wind Tunnels

Low-Speed (M < 0.3) Wind Tunnels

- The velocity of the air in a wind-tunnel's test section is measured either (i) by a pitot-static tube placed in the test section or by (ii) two static ports, one in the settling chamber and one in the test section. The second method has the advantage that static ports do not intrude into the test section and therefore are less likely to interfere with the mounting of a model to be tested.

- Assuming the incompressible flow, applying eq(3.2) yield:
  \[ V_1 = V_2 \frac{A_2}{A_1} \]  
  (3.13)

- Substituting eq(3.13) into eq(3.4) and rearranging:
  \[ P_1 - P_2 = \frac{D}{2} \left( V_2^2 - V_1^2 \frac{A_2^2}{A_1^2} \right) \]
  \[ V_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho \left[ 1 - (A_2/A_1)^2 \right]}} \]  
  (3.14)  
  Wind Tunnel Test Section Airspeed
Example A-1-4
(Low-Speed Wind Tunnels)

Example 3.5

A low-speed wind tunnel, that is similar to the one shown in Fig. 3.8, has a settling chamber with cross-sectional area of 10 m² and a test-section with cross-sectional area of 1 m². When the wind tunnel is run in standard sea-level atmospheric condition, a manometer connected between static ports in the walls of the settling chamber and the test section, as shown in Fig. 3.8, has a difference in the heights of its fluid columns of 50 cm. What are the maximum test-section velocity (magnitude) and the mass flow rate through the test section of this wind tunnel? Also determine these conditions.

Solution (3.5)
Airfoils (1)

Flow past an Airfoil

- The stream tube (flow past an airfoil) upper & lower surfaces, under continuity, will create the surface velocity difference (acceleration/deceleration). As a result, the static pressure difference between upper & lower surfaces will generate the "lift."
- Also, due to the presence of "viscosity," a thin layer of non-uniform velocity near the surface (called, the "boundary layer") is formed. Because of the Newtonian fluid shear stress, this will cause the surface "skin friction."

Pressure & Shear Stress Distribution on the Surface of an Airfoil

- Two different types of surface distributions: (i) pressure (force \perp\) to each local surface per unit area: normal stress distribution) and (ii) shear stress (force \parallel\) to each local surface per unit area: shear stress distribution) will contribute to a total aerodynamic force of a given airfoil shape.
- The surface static pressure distribution: (i) arrows drawn outward from the surface indicate pressures lower than freestream static pressure and (ii) arrows drawn inward to the surface indicate pressures higher than freestream static pressure.
Angle of Attack

- The total aerodynamic force can be resolved into a component \( \perp \) to the freestream (called, the "lift" of airfoil) and a component \( \parallel \) to the freestream (called, the "drag" of airfoil).
- An airfoil "chord line" is defined as a straight line, connecting between the leading edge (l.e.) and trailing edge (t.e.) of an airfoil.
- Now, the total aerodynamic force can also be resolved into a component \( \perp \) to and \( \parallel \) to the chord line (called: the "chordwise force" and the "normal force" of airfoil, respectively).
- The angle between the chord line and the direction of freestream is called the angle of attack (AOA, or \( \alpha \)) of an airfoil.

Relationship between Normal Force & Lift

- The net normal force on a portion of the airfoil surface is the pressure on that portion multiplied by its area. The airfoil surface is, in general, not parallel to the chord line. Thus, if \( ds \) is the length of an infinitesimally small portion of the surface and \( dx \) is the length of the component of \( ds \) along the chord line, the contribution of its surface normal force to the total force normal to the chord line for an airfoil is:
  \[
  dn = Pds \frac{dx}{ds} = Pdx \quad (3.15)
  \]
  Therefore, the magnitude of the total normal force on the airfoil is:
  \[
  n = \int_0^c (P_l - P_u)dx \quad (3.16) \quad \text{Total Normal Force of an Airfoil (per unit span)}
  \]
- The lift on the airfoil is the component of normal force \( \perp \) to the freestream:
  \[
  l = n \cos \alpha \quad (3.17) \quad \text{Total Lift of an Airfoil (per unit span)}
  \]
Boundary Layer & Skin Friction Drag

Boundary Layer

- The region next to a body surface in which the flow velocities are less than the freestream velocity is known as the boundary layer. It is usually a very thin layer, and caused by the presence of a viscosity.

Viscous Effects in the Boundary Layer

- The surface attached viscous flow produces viscous drag, called "skin-friction" drag. Skin friction drag typically comprises about 50% of the total drag on a commercial airliner at its cruise condition. One important factor to reduce aircraft’s skin friction drag is to minimize the surface area that is "exposed" to the air (wetted area).
- Boundary layer, depending on the surface shape, will typically start from "laminar" and then "transition" to "turbulent." The boundary layer thickness, as well as the amount of shear stress (skin friction) will depend on the type & behavior of the boundary layer. The design of shape (airfoil), therefore, will have a major impact on the behavior of the boundary layer.

Wall Shear Stress

- The surface shear stress, due to the presence of the viscosity, can be given by:

$$\tau = \mu \left( \frac{dV}{dy} \right)_{y=0} \quad (3.18) \textbf{ Wall Shear Stress}$$

- $\mu$: viscosity of a fluid
- $y$: direction $\perp$ to the body surface
Skin Friction Drag

- The skin friction drag for a body is given by:
  \[ D_f = \int_0^{S_{\text{wet}}} \tau ds \]
  \( S_{\text{wet}} \): total wetted area of the body

- The skin-friction drag is often expressed as a dimensionless coefficient:
  \[ C_f = \frac{D_f}{q_\infty S_{\text{wet}}} \]  \( (3.19) \)  \textit{Skin Friction Drag Coefficient}

Pressure Drag

- When the boundary layer "separates" from the surface of the body, its pressure is generally less than or equal to freestream static pressure. The difference in pressures at the front and rear of the body produces a net force in the drag direction, called \textit{pressure drag}.

Reynolds Number

- A non-dimensional parameter to quantify magnitudes between momentum & viscosity:
  \[ \text{Re} \equiv \frac{\rho V x}{\mu} \]  \( (3.20) \)  \textit{Reynolds Number}

Solution (3.6)