Lab 1: Experimental Error

Description

You will learn what error in measurement is, how to quantify it, and how to reduce it. You will learn the different kinds of errors and their effect. Finally, you will explore different kinds of plots and the appropriate formatting and methods for plotting.

Equipment

- Ruler
- Meter stick
- Rubber ball
- Bubble level
- Tape (optional)

Introduction

Please read the Error in Measurement - Theory section before beginning this lab.

In the first part of this lab, you will be looking at a source of systematic error. Most systematic errors can be identified and corrected or accounted for. One source of systematic error comes from the human factors that come into play in taking measurements. For example, if the experimenter needs to read a meter stick, the angle at which they view the stick can affect the measurement. If the angle they are viewing the measurement from is always to one side, then they will always read the measurement high or low, thus this is a systematic error. This can be easily fixed by being very careful about how you read the measurement.

Another example of systematic error is an experimenter needing to take a measurement in reaction to an event. If a measurement needs to be taken at a specific time interval or when a trigger is set off (in this lab, when your lab partner lets go of the ruler) then the reaction time of the experimenter will affect the results. This systematic error will be directly affected by how tired the experimenter is as well as how long since they have eaten etc. For this reason it is a good idea to eat before coming to lab!

Some Systematic Errors in Aviation

An obvious application of systematic error, manifested by your reaction time, is encountered in piloting your aircraft on the ground and in the air. From your study of kinematics, you know that if you are traveling at a constant speed "v", then a distance traveled (displacement) is given by:

$$\Delta x = v \cdot \Delta t$$  \hspace{1cm} (1)

where $\Delta t$ is an elapsed time. In the context of this experiment, decisions made in the cockpit can be delayed by the pilot’s reaction time $\Delta t$, during which an aircraft traveling at constant speed...
v will travel some distance $\Delta x$. After completing this lab, you will know your reaction time $\Delta t$, and will therefore be able to calculate the distance your plane travels during your reaction time.

In addition to human reaction time, various components of your aircraft require regular test and calibration. Among these are the following, according to the College of Aviation:

- The altimeter is always set before takeoff to an altimeter setting. It allows a traditional altimeter to correct for non-standard pressure. In reality, the local barometric pressure is taken and corrected for the known elevation of the altimeter/barometer.

- There are three gyros on the airplane. Two run off of a vacuum pump driven by the engine. It pulls air across a spinning gyro to help with attitude (pitch and roll) and another vacuum gyro displays heading (the compass has errors in accel/decel and turns).

- There is a VOT (VOR test) that can be done on the ground. Some airports have a VOT transmitter. You tune the VOR navigation receiver to a frequency. The needle will turn to a certain heading (it is plus or minus 4 degrees) this calibrates the VOR receiver. This only needs to be done if flying IFR and only once every 30 days. An in-flight test is possible.

- The airspeed indicator reads indicated airspeed (IAS). This is slightly different than calibrated (actual) airspeed. The difference between the two is tested and usually listed in the pilot operating handbook (POH). As one climbs, the change in temp and altitude cause a disparity between IAS and True Airspeed (TAS). The lower pressure overcomes the density increase attempted by colder temps at altitude, so as one climbs, a constant IAS corresponds to a higher TAS. Generally, if there are no compressibility issues (because of flying just under Mach 1), at 42,000 feet, TAS is exactly twice as much as IAS. That is, 250 KIAS = 500 KTAS. This can be calculated on a circular slide rule (E6-B) for any IAS, temp, and pressure altitude.

Thus, taking data (testing) and calibrating various functions of your aircraft are a regular part of pilotage and aircraft maintenance and safety.

The second part of this lab will deal with random errors and proper graphing techniques for functions. Graphs are a common way to present information in a concise, compact format, and are commonly encountered in aircraft operations and maintenance manuals. Knowledge of how to properly construct and interpret graphs is therefore essential to students in their major field of study, as well as in this Physics class! In this lab, we will review the construction of graphs, gather simple data, plot data on these graphs and, along the way, explore the concept of random error. As discussed, there exists two types of error, namely systematic (or bias or calibration or offset) error and random error. The former can be virtually eliminated by a careful calibration of measurement devices, but the latter can only be minimized, never entirely eliminated.

### Functions and Graphical Structure

At its best, a graph allows a user to ascertain the behavior of a dependent variable as a function of one (or more) independent variables. As you may recall from prior or concurrent math classes, an independent variable may assume any value, and these values constitute the domain of a function. The function itself is the dependent variable, because its value depends upon the value(s) of the independent variable(s). Values of the dependent variable constitute the range of the function. One
way to visualize the workings of a function is as a "black box" or a "map" that for each value of the input, assigns a particular value to the output. If you let "x" represent an independent variable and "y" the dependent variable, then the process of evaluating the function to produce a single, output value may be represented as shown in Figure 1.

![Figure 1: The operation of a function.](image)

Where the notation f(x) is shorthand for some rule that assigns each value of x to a particular value of y. To generate a graph, we can plot y versus x for a number of different values of x. As a simple example, consider two different values of x: x_1 and x_2 and the corresponding values of y: y_1 and y_2. We assign the single independent variable’s values to the horizontal "x" axis and the dependent variable’s values to the vertical "y" axis. Remember that axes are really just number lines, on which you can plot points. However, since we want to observe the interaction between the independent and dependent variables, we will plot points at the intersections of these data rather than just "on" the number lines themselves, as shown in Figure 2.

![Figure 2: The components of a simple x-y plot.](image)

Required elements are shown in Figure 2 in **bold**. The x_{dn} and y_{dn} markers along the axes indicate the major divisions on the x and y axes, respectively.

Several important points to keep in mind when constructing graphs are:

- In physics classes and labs, use SI (metric, meter-kilogram-second system) units unless specifically directed to use other units;
- You are expected to draw straight lines for the axes, and indicate divisions appropriately (and usually, evenly spaced);
• Make your graphs as large as possible. This will make it easier for you to plot points, and easier for a user to read;

• You are expected to be careful when plotting points, such as the coordinate pairs plotted at points $P_0$ and $P_1$;

Sometimes, the input variable is a time, \( t \), and the output variable, \( y(t) \) is the value of something at the time \( t \). We can then construct a graph of \( y \) versus \( t \) for a selection of times \( t \). For example, suppose \( y(t) \) is the cylinder head temperature at time \( t \). In terms of your situational awareness in the cockpit, consider which would be more valuable, in terms of conveying information to you, the pilot, as quickly as possible: a red LED display or analog gauge display of the (current) cylinder head temperature, or a graphical display of cylinder head temperature as a function of time? If the latter enhanced your functionality in the cockpit by enabling you to make faster decisions, then the graphical display would be preferable to an output display of a single temperature. From the graph, you can determine not only what your current temperature is, but also what past values were, and easily detect trends in the temperature information—the temperature is holding steady, is increasing, or decreasing. For example, suppose the temperature is increasing, and you are flying a single-engine plane. You would like to be able to judge whether you can make the nearest airport before the engine red-lines at around 500F. To make such a judgment, you would not only like to know the rate of temperature increase (i.e. the slope of the graph) but also whether or not the rate itself is holding steady or going up or down (i.e. whether the slope is constant, increasing or decreasing). Based on this information, you could plan ahead in order to make your interaction with the ground as pleasant as possible.

This, then, is the justification for using graphical methods to convey information. The information can be generated from a mathematical relationship, but often (like our temperature gauge example) is generated from sensors or laboratory equipment. In either case, the user or reader can quickly form a mental summary of what is going on.

In another situation applying to flight, graphs are used to calibrate flight equipment. For example, consider that the input value \( x \) could be an encoded voltage input from, say an engine-mounted cylinder-head temperature sensor. In this case the black box/map discussed above would be your avionics, and the output would be the cylinder head temperature, as displayed on your cockpit instrumentation after passing through the avionics (basically some electronics). The temperature displayed depends upon an encoded voltage from the sensor, so that voltage is the independent variable and temperature is the dependent variable.

In this example, voltages would go on the horizontal axis, since they are the independent variable. The horizontal axis label would therefore be Cylinder Head Sensor Voltage [V] or something similar. The vertical axis label could be Actual Cylinder Head Temperature $^\circ$F. This is shown in Figure 3.

Very often, data sets will show regular patterns on your graph, such as appearing to fall on a line or following a smooth curve. In these cases, the data can usually be summarized by a mathematical formula or model of the data. A good model fits your data and at the same time reflects the physical processes going on. For example, for the temperature sensor calibration curve above, you may expect the sensor to give a voltage that is proportional to the temperature. In that case, you would have a straight line. However, the sensor may well deviate from proportional behavior, in which case the calibration curve may be more complicated, and a straight-line model no longer suffices.
The simplest mathematical form to model is the straight-line or linear relationship. In other words, the dependent variable \( y \) depends on the independent variable \( x \) as follows:

\[
y = mx + b
\]  

which will form a straight line (of slope \( m \) and \( y \)-intercept \( b \)) on your graph. Due to the presence of measurement uncertainty, most of the points will not fall exactly on the line, but will be clustered around the line, in a long narrow cloud. The smaller the measurement uncertainty, the narrower the cloud of points.

For linear relationships, the best fit line to the data can be estimated by eye and drawn by hand. When you do this:

- Don not "connect the dots" of your data set.
- Use a ruler or straight edge to draw a straight line through the data.
- Try to balance the cloud of data points about the line. Unless there are substantial deviations from a linear form, this means that you will have a roughly equal number of data points are above the line as below the line.
- Use additional information when available. For example, if you know (your model requires) that the line should pass through the graph’s origin at \((0,0)\), then make sure your drawn best-fit line passes through the origin while continuing to address the point made in the preceding bullet.

Using our temperature sensor example one last time, and plotting more points and a best-fit line, we obtain Figure 4.

The distance between the an actual data point, and the corresponding \( x \)-location on the best fit line/curve is called the residual. In Figure 4, the residual between one of the data points and the best fit line is indicated. The average size of all the residuals is a measure of the uncertainty in
your data. (In fact the uncertainty is formally defined to be approximately equal to the geometric average of all the residuals.) Although, in this course you will only do "by-eye" fits to data, there is a formal way of getting the best fit curve. It is called "least-squares" fitting. In least-squares fitting, the total distance of the data points to the line/curve is minimized. Different lines/curves are tried until the one with for which the sum of the squares of all the residuals gives the smallest value. This is then the best-fit line or curve. Many computer programs like Microsoft Excel, MatLab and Mathematica have least-squares fitting routines coded into them. You can therefore use such programs to perform fits to your data. (Excel is currently difficult to use for fits other than straight-line fits.)

**Procedure**

**Part 1:** Systematic error due to human reaction time

In this experiment, you will be using a ruler to measure your reaction time. Your lab partner will hold the ruler vertically so that it hangs between your thumb and forefinger which should not touch the ruler. Your partner will then drop the ruler and you will try to catch it between your thumb and forefinger, without moving your hand up or down! By noting how far the ruler fell before you could catch it, you can calculate your reaction time. See Appendix A for an explanation of the kinematics and how to calculate your reaction time. By comparing your results to those of your classmates, you may be able to see a correlation between reaction time and external factors such as the amount of sleep a person has had, or the time since they last ate.

1. Record your current conditions: Age, sex, how long since you last slept, how long since you last ate, how you feel. Include any other factors that you think will affect your reaction time.

2. Have your partner suspend a ruler between your thumb and forefinger, so that the point where the ruler reads zero centimeters is centered between your thumb and forefinger. Your partner should let go of the ruler and let it drop between your thumb and forefinger. As soon as you see the ruler start to drop, try to grab it with your thumb and forefinger. Read the centimeter scale where your thumb and forefinger grabbed the ruler and use the formula above to calculate your corresponding reaction time.
3. Calculate your reaction time for this trial. You may wish to refer to Appendix A: kinematics to do this. Does this time seem reasonable?

4. Repeat this measurement 30 times and record them in a table in your lab book.

Part 2: Random error and graphing

1. Place a meter stick vertically next to a table or next to a wall with the metric side visible; you may wish to use tape to secure the meter stick to a surface, and you may use a bubble level to ensure that the meter stick is truly vertical.

2. Place the ball next to the meter stick. With the bottom of the ball at an arbitrary height, record this height and drop the ball. To the best of your ability, observe and record the height to which the ball bounces or rebounds. This should be the height of the bottom of the ball at the top of the rebound. In a table, record this data in your lab book and indicate which of the two heights recorded in each trial is the independent variable (the one you can control) and which one is the dependent variable. Repeat this until you have a total of 5 heights, with one measurement each. These will be your single measurements.

3. Make a new table and re-measure the rebound height 5 times for each drop height. You should have a total of 15 measurements. When each set of trials is complete, compute the average bounce height for each initial height. These will be your average measurements.

4. Choose three initial heights that are different from what you have used previously. For each one, drop the ball three times and record initial and final heights in a new table. Leave a column in your table for the average of these three and the expected height. You will use these measurements to test your model/best fit line.

Analysis

Part 1

1. Determine the average of your 30 trials and record it in your lab book. The average of a set of numbers isn’t always guaranteed to be the most likely, and hence representative, outcome. For example, skewed distributions feature an average value which is offset from the most likely (or most frequent) outcome. In these cases, the statistical mode, the value of the reaction time (bin) which occurs most frequently, is a better characterization of your reaction time than the mean, or average.

2. Make a frequency plot, also known as a histogram, of your trials. You may either hand draw the plot or use a computer. If you hand draw it, you should use a ruler, if you use a computer then print it out and permanently glue it into your lab book. Be sure all graphs take up at least half the page. For this part, see Appendix B.

3. Is your plot Normal (a.k.a. Gaussian, or bell-shaped) or skewed? Find the mode of your data? Is the mode (approximately) the same as the mean?

4. Can you explain any outliers, that is, any points which lie well away from the remainder of data? In general, do you think your plot is representative of your performance?
5. The histogram you generated above gives you an easy way of identifying the uncertainty in each individual measurement of the reaction time. Basically, the uncertainty is given by the width of the histogram divided by two. You can read this directly off of the histogram. (Ask your instructor to show you how to eyeball this!) You can also use the fact that the width/2 of a Normal distributions given by the following formula and is known as the Standard Deviation, \( \sigma \):

\[
\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (t_i - \bar{t})^2}
\]

Where \( t_i \) are the individual response time measurements and \( \bar{t} \) is the average response time. \( N \) is the number of measurements (30 in this case).

6. What is the uncertainty in your measurements, using either the definition of the standard deviation above or by eyeballing the width of the histogram and dividing by two.

7. Is the uncertainty in the individual measurements you found above larger or smaller than the scale-reading uncertainty on the ruler? If so, why?

Part 2

1. Using your single measurements, construct a graph of the data points. Be sure to include all necessary components of a graph! Draw a best-fit line on the graph.

2. On the same graph, add the average measurement for each height using a different color. Draw a best fit line for this data.

3. For the heights in step 4, predict the bounce heights using the two best-fit lines you have just plotted. Do this by the following procedure:

   - Starting with the initial height, draw a vertical line from the horizontal, "initial height" axis upwards for each of the initial height values; these lines should cross your best-fit lines.
   - Go to the point where the vertical line crosses the best-fit line.
   - Draw a horizontal line from this point to the vertical "bounce height" axis and record this value in your table it is your predicted bounce height using "single measurement" data.
   - In your table, compute the residuals between the actual bounce heights and the predicted bounce heights by subtracting one from the other for each trial. Square the residuals and add them.

Please answer the following questions.

1. One way to minimize the effect of random error/uncertainty on a measured value is to take multiple readings of the dependent variable for the same independent variable value. This is what you did in step 3, by performing multiple trails using the same initial height, then averaging the resulting bounce heights. Does this procedure actually affect the best fit line? Explain.
2. Using your averaged data, consider the statistical range in your observed results. Does range show any correlation with initial drop height? If so, discuss or describe how range varies with initial drop height.

3. Describe some sources of random error that you observed in the experimental procedure:

4. Refer to your best-fit line drawn using the averaged data set. Pick two points on this line. The points should be fairly far apart, perhaps towards the ends of the line you have drawn on your graph. Determine the "x" coordinate of each point by drawing a vertical line from the point down to the "x" (horizontal) axis. Determine the "y" coordinate by drawing a horizontal line from the point to the "y" (vertical) axis. Record these coordinates in your lab notebook (note that it does not matter which point you designate to be "1" or "2"). Compute the equation of your best fit line in the form $y = mx + b$. 
Appendix A: Kinematics

From kinematics, the displacement as a function of time for an object experiencing a uniform acceleration is:

$$y = v_0 t + \frac{1}{2} a t^2$$  (1)

where $y$ is the displacement of the object, $v_0$ is the initial velocity of the object, $a$ is the acceleration acting on the object, and $t$ is the time the object is in motion.

In this case, the object we are considering is the folded piece of paper. Because we let go of the paper and allow it to fall, the initial velocity of the paper is zero. However, the paper does experience an acceleration due to gravity as it falls. Thus, Equation 1 reduces to:

$$y = \frac{1}{2} g t^2$$  (2)

where $a$ has been replaced by $g$, the acceleration due to gravity at the surface of the earth.

Thus, if we know the distance through which an object has fallen, we can calculate the object’s time of flight by rearranging Equation 2. To get $t^2$ by itself, divide both sides of Equation 2 by $\frac{1}{2} g$:

$$\frac{2y}{g} = t^2$$  (3)

Thus,

$$t = \sqrt{\frac{2y}{g}}$$  (4)

The values on your paper were calculated using this equation. Each line on the numbered side is 0.25 [cm] thick, and thus each number corresponds to a distance of 1.0 [cm], and by plugging each of these values into Equation 4, we obtain all the times listed in the chart on your paper.
Appendix B: Statistics and Frequency Plots

Descriptive Statistics

The most common descriptive statistics, i.e. the accepted quantitative methods of describing a data set, are:

- The average (or mean): a measure of the central tendency of the data, or the clustering of the data about a single value.
- The standard deviation: a measure of the spread in the data about the mean.
- The median: if a data set is ordered from smallest to largest value, the median is the value in the center (for an odd number of values) or the average of the two middle values (for an even number of values)
- The mode: the value which appears most frequently

In addition, there are two less common but important ways to characterize the shape of a data distribution, namely:

- The skewness: a measure of the degree to which a data set or distribution is non-symmetric; the most common outcome is that the distribution has a long tail on one side of the center.
- The kurtosis: a measure of the sharpness or flatness of the distribution, as compared to a bell-shaped (Normal or Gaussian) curve.

Frequency Distributions and Plots

The frequency of data answers the question “how many times does this value (or similar values) show up in my data set”. Because it does not make sense to talk about the frequency of a single data point, in general (think about it: the frequency of a unique data point is exactly one (1.0), if that particular value, out to some number of decimal places, only appears once), we can “bin” similar data points together to see how often a group of data, with similar values, shows up within the overall data set.

As an example, consider a game where you throw pennies into a row of cups, with the intent of getting the penny in the middle cup as often as possible: the person with the most pennies in the middle cup wins. The cups are lined up as shown below:

![Figure 1: Throwing cups.](image)

After throwing coins, you count the number of pennies in each cup. Stacking the coins, you observe the outcome shown in Figure 1.

In this example, you have one penny in cups 1 and 2, two pennies in cup 3, four pennies in cup 4 (the middle cup), three pennies in cup 5, one penny in cup 6, and you missed cup 7 entirely.

Each cup constitutes a “bin” in the statistical sense. The frequency is just the number of pennies in each bin. The frequency distribution is the outcome of the game, considering all the cups (the “entire ensemble” of data). You can construct a frequency table easily, as shown below.
Table 1: The frequency distribution

<table>
<thead>
<tr>
<th>Bin (cup number)</th>
<th>frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
</tr>
</tbody>
</table>

You can also easily construct or draw a frequency histogram, as shown in Figure 3 (Microsoft Excel was used to construct this histogram using the “Graph Wizard” menu).

The difference between this simple gaming example and your data set is that the number of bins is not pre-defined (as in the game, where you used a certain number of cups). You will need to come up with logical choices for how to determine the number of bins to use and the width (in time, since you are measuring times) of the bins. After you determine these two values, distributing your data into the bins is elementary.
Practical Considerations for your Frequency Plot

One of the hardest things to determine is how many bins to use to describe your distribution. A common rule of thumb to determine the number of bins you should use is:

\[ N = 1 + 3.3 \log n \]

where \( N \) is the number of bins to use and \( n \) is the number of your data points, in this experiment, \( n = 30 \). \( \log \) is the log base 10 on your calculator, accessed by pressing the [LOG] button. Notice that obtaining an integer number (2, 3, 4, 5) of bins is unlikely, so round up the number you obtain using this technique to get an integer number of bins.

In this experiment, \( n = 30 \). Evaluating the above expression, \( N = 5.87 \). You should round this up to six (6) bins.

To determine the width of the bins, refer to Table 2 and determine your smallest and largest reaction time. Call these \( t_s \) and \( t_g \), respectively. The data range, \( R \), is defined as:

\[ R = t_g - t_s \]

The width of each bin is just the data range divided by the number of bins you are using, or:

\[ W = \frac{R}{N} \]

Example

Let us suppose that your smallest value from Table 2 was 0.096, and your largest value was 0.228. Then the data range is \( R = 0.132 \) [s], and the width \( W \) of your bins is just 0.022 [s]. On a practical basis, you may wish to round this value up or down slightly to make it easier, e.g. you may wish to round this up to 0.025 [s] or down to 0.020 [s]; in some cases, this may alter the number of bins required to contain all your data. So be it.

For this example, you would construct your bins as shown in Table 2, using the rounded value of 0.025 [s] for \( W \).

<table>
<thead>
<tr>
<th>Bin number</th>
<th>Lower bin boundary</th>
<th>Upper bin boundary</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.095</td>
<td>0.120</td>
</tr>
<tr>
<td>2</td>
<td>0.120</td>
<td>0.145</td>
</tr>
<tr>
<td>3</td>
<td>0.145</td>
<td>0.170</td>
</tr>
<tr>
<td>4</td>
<td>0.170</td>
<td>0.195</td>
</tr>
<tr>
<td>5</td>
<td>0.195</td>
<td>0.220</td>
</tr>
<tr>
<td>6</td>
<td>0.220</td>
<td>0.245</td>
</tr>
</tbody>
</table>
Suppose you examine your data in Table 2, and the first two data are reaction times of 0.172 [s] and 0.133 [s]. The first data point would go into bin 4, because it falls between reaction times of 0.170 and 0.195 [s], while the second data point would go into bin 2, because it falls between reaction times of 0.120 and 0.145 [s].

After distributing your data into the six bins, you can construct or draw a histogram similar to Figure 4 for your data set.

Let us return to the gaming example. Suppose you toss 17 pennies into the cups, and you obtain the fairly bell-shaped curve below. In this case, your computed average (or mean) value will fall into the center cup, number 4. Since the mean and the mode (the bin into which the greatest number of pennies fell) occur in the same cup, it is fair to say that your distribution is bell-shaped, Normal, or Gaussian (three terms for the same thing).

![Figure 4: Example Gaussian distribution with mean=mode.](image)

In the case of a skewed distribution, as shown below, the mode is in bin 6.

![Figure 5: Example of a skewed histogram with mode=6.](image)