Resistance

Objectives

Build a Wheatstone Bridge circuit to measure the resistance of short lengths of wire. You will investigate the dependence of the resistivity on wire length, radius and material type.

Equipment

- Small Breadboard
- Assortment of jumper wires
  (Note: The 3 items above are already installed on the Radio-Shack "Electronics Learning Kit.")
- Several known resistors R_N: 1.0, 2.2, .7, 10 Ohm
- Several known resistors R_M: 1k, 4.7k, 10k, 47k, 100k Ohm
- Rheostat $R_a \approx 5-400\Omega$
- Multimeter
- Ruler
- Micrometer
- Test lead with alligator clip attached
- Copper or brass wire cut to the following lengths:
  - 1 long (~50 cm) segment
  - 8 short segments all of the same length (~10 cm)
- 1 long (~50 cm) steel wire segment

Introduction

Electrical Resistance is an interesting property of conductors. It is not a fundamental property like mass but an emergent property that depends on the type and physical arrangement of the atoms in the conductor. Resistance measurements therefore, in some sense a probe of the structure of conductors. However, the resistance of an object depends not only on the material it is made from, but also on its size and shape, factors which must be accounted for if we want to interpret the resistance as a property of the material itself. In this lab, we will investigate the dependence of the resistance of wires on their length, cross-sectional area and wire material.

Unfortunately, the resistance of conductors is so low that the resistance of a short length of wire is so small that it is not easily measurable by direct application of Ohm's Law. Recall, that Ohm's Law states that voltage across a conductor is proportional to the current through the conductor. The constant of proportionality is called the resistance, R.

$$R = V / I \tag{1}$$

For conductors with resistances on the order of several Ohm's or kilo-Ohms, we could simply apply a voltage across the conductor and measure the resulting current. Then, the ratio of the voltage to the current would give us the resistance. In principle this method could also work for the very small resistances presented by short lengths of wire (typically below several hundredths of an Ohm). In practice however, this is difficult to do. Small voltages (on the order of a single volt, say) lead to very large currents (hundreds of amps!) through such small resistors. Such currents are difficult to produce and measure. Attempting to draw a current of several hundred Amps from a small battery or bench-top power supply would certainly over-tax the device. For this reason, most commercial Ohm-meters and multimeters (which must themselves source the current through the resistance) cannot accurately measure very small resistances.

One way of measuring the small resistances inherent in short segments of wire is to use a so-called bridge circuit. The classic bridge circuit, known as the “Wheatstone Bridge” is shown below. While there are more complicated bridge circuits that would allow us to obtain even more accurate results (e.g. by using a bridge circuit known as a “Kelvin Double Bridge”) the Wheatstone Bridge is comparatively simple and accurate enough the purposes of our measurements.

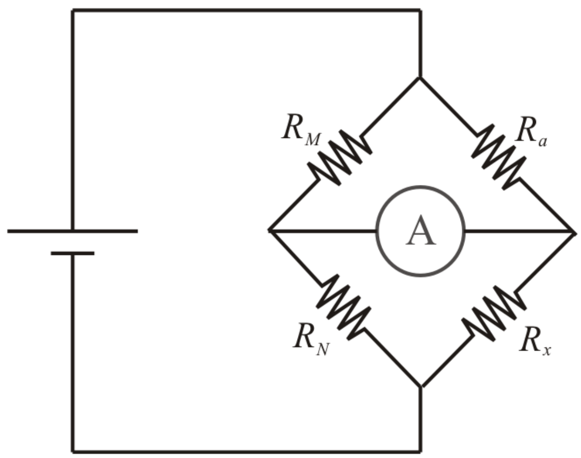

Figure 1: A Wheatstone Bridge

The three resistors R_M , R_N and R_a are known while the resistance R_x is to be determined.

For practical purposes, it suffices to know that the way this circuit is generally used is to have R_a be an adjustable resistor which is varied until no current flows through the galvanometer, $I_g = 0$. The bridge is then said to be “balanced.” It is shown in the appendix that this occurs when the following relationship between the resistors holds

$$\frac{R_x}{R_a} = \frac{R_N}{R_M}. \tag{2}$$

Sometimes, R_M or R_N are also allowed to be variable in order to make it easier to balance the bridge (i.e. achieve the condition described by Equation 2). When the bridge is balanced, we can calculate R_x by solving the Equation 2 for R_x . In other words,

$$R_x = R_a \frac{R_N}{R_M}.$$

So this is the procedure we will be adopting. Balance the bridge, measure R_a , R_M and R_N . Then calculate R_x from the equation above. We will then use the resistance data we acquire to test the following model for wire resistance

$$R = \frac{\rho L}{A} \tag{3}$$

where L is the wire length, A is the wire cross-sectional area ($A = \pi r^2$ where r is the wire radius) and ρ is a material constant known as the *resistivity*.

> A Note on the Wheatstone Bridge and Strain Gauges
>
> The Wheatstone bridge is particularly good for measuring small differences in resistance between a comparison "standard resistor" R and a resistance under test with a slightly different value $R+\varepsilon$. In one common application, The circuit has three identical standard resistors R and one resistor that has a slightly higher or lower resistance, $R+\varepsilon$, where ε may vary. If ε is non-zero, the right-hand leg of the circuit will have a slightly higher or lower resistance than the left hand leg. The two paths will be unbalanced and a current will flow through the ammeter. Provided ε is small, the current through the ammeter is small and can therefore be measured very accurately with a sensitive galvanometer. The resulting current is (to first order) proportional to ε. This is the principle behind the strain gauge. The strain gauge is merely a piece of foil that is glued to an object that deforms. As the object deforms and the foil deforms with it, the resistance of the foil changes slightly. A Wheatstone Bridge is then used to measure this small change in the resistance. The current through the ammeter is (to first order) proportional to the strain.

Procedure

Preliminary Note: If you are unsure of how to configure the multi-meter to measure DC current, please ask your instructor for help. Also, if you have never used a breadboard before, please ask your instructor to explain how to insert components into the breadboard and obtain an explanation of how the connections in the breadboard are laid out. Your instructor will explain the resistor color coding scheme so that you can read the resistor values from the colored bands on the resistors.

1. Using the multi-meter configured to measure resistance, measure and record the resistances of the resistors R_M and R_N. Make sure you do this with the resistors *disconnected* from the circuit. Use the resistors and the breadboard provided to build the Wheatstone Bridge Circuit according to Fig. 1 using the resistor values given above. As your ammeter "A" use the multi-meter configured to measure DC current. The wire segments whose resistances you are to measure, take the place of resistor R_x. Using the bridge circuit you constructed, measure and record the resistance of each wire segment. You will do this for each wire segment as follows:

    a. Balance the bridge circuit by adjusting R_a until no current flows through the ammeter. If you need to move the rheostat to the end of its range to balance the bridge, or you cannot balance the bridge at all, change resistors R_M and/or R_N until the bridge is balanced with the rheostat somewhere in the middle of its range.
    b. Disconnect the rheostat R_a from the circuit without moving the slider and use the ohm-meter to measure and record it's resistance R_a.
    c. Calculate the resistance of the wire segment R_x from Eq. 2 above.

2. Prepare to take measurements by setting up a table in your lab book with the following column headings: Wire Material, Wire Length, L (mm), Wire Diameter, d (mm), R_M (Ω), R_N (Ω),

R_a (Ω), R_x (Ω), $\rho\,(\Omega\mathrm{m})$, Comments. In the column for comments, you will register any comments you may have about the individual measurements, noting any anomalies, etc. Make sure you record at least the dominant uncertainties with every measurement. (If you are unsure which uncertainties are likely to dominate, go ahead and record all uncertainties!)

3. Measure the resistance of your long copper or brass wire by plugging one end of the wire into the breadboard (at the junction of the ammeter and R_a) as shown in Fig. 1. Connect the other end to the test lead with the alligator clip. Balance the bridge using the rheostat. Disconnect the rheostat and measure its resistance. Use Equation 2 to calculate the resistance of the long copper wire. Record the resistance and the other relevant parameters in your table.

4. Now shorten the effective length of the copper wire by moving the alligator clip. Balance the bridge and calculate and record the resistance of the shortened wire. Do this about 8 times, each time with a shorter effective length, ending with a very short length of copper wire.

5. Repeat step 4 using the steel wire in place of the copper/brass wire.

6. Construct a graph of resistance R_x versus wire length, L for both steel and copper wire and be sure to include error bars for each point. Note that the *y*-intercept gives you the resistance of the test lead.

7. Now measure the resistance of the short lengths of copper wire (~20 cm) starting with a single short length of wire and adding more segments *in parallel* until you are measuring the resistance of all 8 short lengths of wire in parallel. Construct a graph of Resistance R_x versus total wire cross-sectional area A (sum of the cross sectional areas of all wires in parallel).

8. For each of the individual resistance measurements for copper wire that you made above (steps 3, 4 and 7) calculate a value for the resistivity ρ of copper via

$$\rho = \frac{R_x A}{L}.$$

   and enter it into the table you constructed above.

9. Make a histogram of all the resistivities ρ of copper that you found in the last step and calculate the mean $\bar{\rho}$ and standard deviation. Use the standard deviation as a measure of the uncertainty, $\Delta\rho$, in individual measurements of ρ. If $\Delta\rho$ is the uncertainty in each individual measurement of ρ, find the uncertainty $\Delta\bar{\rho}$ in $\bar{\rho}$. Since the mean $\bar{\rho}$ is your best estimate of the resistivity of the copper in your wires, you should quote your final result for the resistivity of your copper as

   Resistivity of copper in my wires = $\bar{\rho} \pm \Delta\bar{\rho}$

10. Calculate the resistivity of steel from the slope of the trend of the steel wire.

Questions

Q1. What was the largest source of uncertainty in the resistance measurements and how could this have been improved?

Q2. What is likely to be the single largest source of systematic error? Justify your answer carefully.

Q3. Does your data support the model in Equation 3? To what extent could there be deviations of the true behavior from the model that are not ruled out by your data?

Appendix

Equation 2 may be shown to be true as follows
(material adapted from http://en.wikipedia.org/wiki/Wheatstone_bridge).

First apply Kirchoff's Rule for currents to the junctions with the galvanometer (leftmost corner of the bridge and rightmost corner of the bridge)

$$I_a - I_x + I_g = 0$$
$$I_M - I_g - I_N = 0$$

Then apply the Loop Rule for voltages to the upper and lower loops of the bridge

$$R_a I_a - R_g I_g - R_M I_M = 0$$
$$R_x I_x - R_N I_N + R_g I_g = 0$$

Since the bridge is balanced, the current through the galvanometer is zero, $I_g = 0$. So, the last two equations can be rewritten as

$$R_a I_a = R_M I_M$$
$$R_x I_x = R_N I_N$$

Dividing these two equations and rearranging gives

$$\frac{R_x}{R_a} = \frac{R_N}{R_M}$$

which is the same as Equation 2.