Summary (Governing Equations) of Thin Airfoil Theory: Unit C-4 (AE301 Hayashibara)

Symmetric Thin Airfoil Solutions

The total circulation around the airfoil is: \[ \Gamma = \int_0^c \gamma(\xi) d\xi = \frac{c}{2} \cos \theta d\theta \]

Solution of the thin airfoil theory is: \[ \Gamma = \alpha c V \int_0^c (1 + \cos \theta) d\theta = \pi \alpha c V \]

From the Kutta-Joukowski theorem: \[ L' = \rho V \Gamma = \pi \alpha \rho _c V_c^2 \]

The lift coefficient: \[ c_l = \frac{L'}{q_c c} = \frac{\pi \alpha \rho _c V_c^2}{\frac{1}{2} \rho _c V_c^2 c} = 2\pi \alpha \]

The zero lift angle of attack: \( \alpha_{L=0} = 0 \)

Moment at leading edge: \[ M'_{LE} = -\int_0^c \xi (dL) = -\rho V \int_0^c \xi \gamma(\xi) d\xi \]

through transformation and integration: \[ M'_{LE} = -q_c c^2 \frac{\pi \alpha}{2} \]

The moment coefficient at leading edge: \[ c_{m,LE} = \frac{M'_{LE}}{q_c c^2} = -\frac{\pi \alpha}{2} = \frac{c_l}{4} \]

The moment coefficient at quarter chord point: \[ c_{m,c/4} = c_{m,LE} + \frac{c_l}{4} = \frac{c_l}{4} + \frac{c_l}{4} = 0 \]

The location of center of pressure is: \[ \bar{x}_{cp} = \frac{1}{4} \frac{c_{m,c/4}}{c_l} = \frac{1}{4} \frac{0}{2\pi} = \frac{1}{4} \]

The location of aerodynamic center is: \[ \bar{x}_{ac} = -\frac{m_c}{a_0} + \frac{1}{4} = -\frac{0}{2\pi} + \frac{1}{4} = \frac{1}{4} \]

Cambered Thin Airfoil Solutions

The lift coefficient: \[ c_l = 2\pi [\alpha - \alpha_{L=0}] \]

The zero lift angle of attack: \( \alpha_{L=0} = -\int_0^\pi \frac{dz}{\pi \theta} \left( \cos \theta - 1 \right) d\theta \)

The moment coefficient at leading edge: \[ c_{m,LE} = \left[ \frac{c_l}{4} + \frac{\pi}{4} \left( A_1 - A_2 \right) \right] \]

The moment coefficient at quarter chord point: \[ c_{m,c/4} = \frac{\pi}{4} \left( A_2 - A_1 \right) \]

Location of the center of pressure: \[ \bar{x}_{cp} = \frac{c}{4} \left[ 1 + \frac{\pi}{c_l} \left( A_1 - A_2 \right) \right] \]

Note: \[ A_n = \frac{2}{\pi} \int_0^\pi \frac{dz}{dx} \cos n \theta \cos \theta d\theta , \] means that . . .

\[ A_1 = \frac{2}{\pi} \int_0^\pi \frac{dz}{dx} \cos \theta d\theta \quad A_2 = \frac{2}{\pi} \int_0^\pi \frac{dz}{dx} \cos 2\theta d\theta \]