(DO-IT-YOURSELF) Review Problem 1-1 (Unit A-1)

Answer the followings (explain in your own words).
(a) List basic dimensions (MLT & F) of SI unit system. Also, what is “kilogram-force”?
(b) List basic dimensions (MLT & F) of U.S. Customary unit system. Also, what is “pound-mass”?
(c) What is the difference between “streakline,” “pathline,” “streamline”?
(d) Is air “always” ideal gas? Are there any “different” types of ideal gas model for air?
(e) What is the difference between “psia” and “psig”? Explain the relationship between them, using $P_{atm}$.

(DO-IT-YOURSELF) Review Problem 1-2 (Unit A-1)

At a point of the surface of a supersonic fighter jet aircraft, the air temperature and pressure are: $-20 \degree C$ and 30 psia, respectively. Calculate the air density at this point, both in SI (kg/m$^3$) and U.S. Customary (slugs/ft$^3$) units.

Hints . . .
• Use conversion: 1 ft = 0.3048 m.
• Use conversion: 1 lb = 4.448 N.
• Use conversion: 1 K = 1.8 $\degree$R.

Answer Keys (No guarantee for 100% accuracy. For your “self-checking” purposes only. Do not “start” from answer keys!)

2.8485 kg/m$^3$ 0.005528 slugs/ft$^3$

(DO-IT-YOURSELF) Review Problem 1-3 (Unit A-1)

A person ‘X’ weighs 200 “pounds” on the earth’s surface.
(a) If this person ‘X’ is located on the moon (1/6 gravity of earth), determine the mass of this person ‘X’ in “pounds.”
(b) If this person ‘X’ is located on the moon (1/6 gravity of earth), determine the weight of this person ‘X’ in “pounds.” A person ‘Y’ weighs 90 “kilograms” on the earth’s surface.
(c) If this person ‘Y’ is located on the moon (1/6 gravity of earth), determine the mass of this person ‘Y’ in “kilograms.”
(d) If this person ‘Y’ is located on the moon (1/6 gravity of earth), determine the weight of this person ‘Y’ in “kilograms.”
(e) Do you think it is a great idea to act like a typical “non-engineering type” person (such as a customer representative of a supermarket), saying “pounds are pounds and/or kilograms are kilograms, what’s the problem?” YES or NO. Explain WHY.

Hints . . .
• Apply $W = mg$ (weight = mass times gravitational acceleration).
• 1 lbm of object weighs 1 lb on earth (at g = 32.2 ft/s$^2$).
• 1 kg of object weighs 1 kgf on earth (at g = 9.8 m/s$^2$).
• “Mass” never change, while “weight” (the “force” developed, due to “mass”) depends on gravitational acceleration.

Answer Keys (No guarantee for 100% accuracy. For your “self-checking” purposes only. Do not “start” from answer keys!)

(a) 200 lbm (b) 33.333 lb (c) 90 kg (d) 15 kgf (e) ?

(DO-IT-YOURSELF) Review Problem 1-4 (Unit A-2)

Answer the followings (explain in your own words).
(a) What is “absolute” altitude
(b) What is “geometric” altitude?
(c) What is “geopotential” altitude? Also explain why we need to employ this, in standard atmosphere model?
(d) What is “gradient region” of atmosphere?
(e) What is “isothermal layer” of atmosphere?
REVIEW PROBLEMS WILL NOT BE COLLECTED or GRADED (all DO-IT-YOURSELF)

(DO-IT-YOURSELF) Review Problem 1-5 (Unit A-2)

At a “gradient region” of the U.S. standard atmosphere model, the temperature is assumed to be a linear function, such as:

\[ T = T_i + a \left( h - h_i \right), \]

where:

- \( h \): a given altitude within the region
- \( T \): temperature at a given altitude \((h)\) within the region
- \( h_i \): the base altitude of the region
- \( T_i \): temperature at the base altitude \((h_i)\) of the region
- \( a \): lapse rate

Starting from the hydrostatic equation with geopotential altitude \((dp = -\rho g dh)\):

(a) Derive an expression of pressure ratio \((p/p_i = ?)\) between a given altitude \((h)\) and the base altitude \((h_i)\) of the region.

(b) Derive an expression of density ratio \((\rho/\rho_i = ?)\) between a given altitude \((h)\) and the base altitude \((h_i)\) of the region.

(DO-IT-YOURSELF) Review Problem 1-6 (Unit A-2)

At an “isothermal layer” of the U.S. standard atmosphere model, the temperature is assumed to be constant.

Starting from the hydrostatic equation with geopotential altitude \((dp = -\rho g dh)\):

(a) Derive an expression of pressure ratio \((p/p_i = ?)\) between a given altitude \((h)\) and the base altitude \((h_i)\) of the layer.

(b) Derive an expression of density ratio \((\rho/\rho_i = ?)\) between a given altitude \((h)\) and the base altitude \((h_i)\) of the layer.

(DO-IT-YOURSELF) Review Problem 1-7 (Unit A-2)

(a) Download and printout standard atmospheric data sheet in SI units (AE301 course web: “NOTES” section). You need to always bring it to the class. This data sheet will frequently be used in the class.

(b) Calculate the standard atmosphere values of \(T\) (temperature: in “\(K\)”), \(p\) (pressure: in “\(N/m^2\)”), and \(\rho\) (density: in “\(kg/m^3\)”) at a geometric altitude of 30 km (start from the sea-level). Check and confirm your results against atmospheric data table. You must perform all your calculations in SI unit system (lapse rate of the first and second gradient regions are: \(a_1 = -6.5 \times 10^{-3} \, K/m \) & \(a_2 = 3 \times 10^{-3} \, K/m\), respectively).

Hints . . .

- Apply standard atmospheric model equations: first gradient region \(0 \Rightarrow 11 \, km\), isothermal layer \(11 \, km \Rightarrow 25 \, km\), and second gradient region \(25 \, km \Rightarrow 30 \, km\) (note: \(30\, km\) is in geometric altitude).
- You must use geopotential altitude (not geometric altitude of 30 km) in your calculation.
- Start your calculation from the standard sea-level values.

Answer Keys (No guarantee for 100% accuracy. For your “self-checking” purposes only. Do not “start” from answer keys!)

(b) \[ \begin{array}{ccc} 231.239 \, K & 1.188 \times 10^5 \, N/m^2 & 0.018 \, kg/m^3 \end{array} \]

(DO-IT-YOURSELF) Review Problem 1-8 (Unit A-3)

Answer the followings (explain in your own words).

(a) What is “incompressible subsonic” flow?

(b) List assumption(s) built into Euler’s equation \((dp = -\rho V dV)\)

(c) List assumption(s) of Bernoulli’s equation \((p + 0.5 \rho V^2 = \text{constant})\) applied on top of Euler’s equation.

(d) What is the difference between “calorically perfect” and “thermally perfect” ideal gas?

(e) What is “isentropic” flow?
Consider the same duct, discussed in class, with an inlet diameter \( d_1 = 10 \text{ m} \) and exit area \( A_2 = 62.832 \text{ m}^2 \).

(a) Air enters this duct with a velocity of 70 km/h (with standard sea-level pressure and temperature) and leaves the duct exit with negligible temperature change at exit. What is the airspeed, in “km/h,” at exit?

(b) Air enters this duct with a velocity of 700 km/h (with standard sea-level pressure and temperature) and leaves the duct exit with 20% lower pressure at exit and 10% increase of temperature. What is the airspeed, in “km/h,” at exit?

Hints . . .
- Is density constant or not constant? Check the Mach number.
- This may **NOT** be isentropic flow (you cannot apply energy equation).

**Answer Keys (No guarantee for 100% accuracy. For your “self-checking” purposes only. Do not “start” from answer keys!)**

(a) 87.5 km/h  
(b) 1,203 km/h

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(a) Consider a small cubic element of air (with the volume: \( dxdydz \)) moving steadily along a streamline. Applying conservation of linear momentum (Newton’s second law of motion: \( F = ma \)) in the direction along the streamline (as shown in the figure), derive Euler’s equation. You should ignore the effects of body forces and viscosity.

(b) Starting from Euler’s equation, derive Bernoulli’s equation along the streamline. You should assume constant density along the direction of streamline.

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(a) For a calorically perfect ideal gas, one can define internal energy and enthalpy (per unit mass): \( e = c_v T \) and \( h = c_p T \).

Starting from the definition of enthalpy, \( h = e + pv \), derive an important relationship between two specific heats:

\[
  c_p = c_v + R
\]

(b) For a calorically perfect ideal gas, one can define a specific heat ratio: \( \gamma \equiv c_p / c_v \).

Starting from the definition of enthalpy, \( h = e + pv \), derive the important relationships between the specific heat ratio, specific heats, and the specific gas constant of air as:

\[
  c_p = \frac{\gamma R}{\gamma - 1} \text{ and } c_v = \frac{R}{\gamma - 1}
\]

Hints . . .
- Start from \( h = e + pv = e + RT \) (note that \( pv = RT \), ideal gas equation of state).
- Differentiate and apply definitions of specific heats \( \left( \frac{dh}{dT} = c_p \text{ and } \frac{de}{dT} = c_v \right) \)

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Staring from the alternative form of the first law of thermodynamics \( \delta q = dh - vdp \) and Euler’s equation \( dp = -\rho VdV \), derive energy equation along the streamline. You should assume that the flow is isentropic, and air is a calorically perfect ideal gas.