Classify the order of each differential equation.

1. \( \frac{dy}{dx} + x^3 y = e^x \)
2. \( \frac{d^3 y}{dx^3} + 7 \frac{d^2 y}{dx^2} - 3 \frac{dy}{dx} + 4 y = \cos x \)
3. \( x^3 dy + y^2 dx = 0 \)
4. \( \frac{d^3 y}{dx^3} + 5 \left( \frac{d^2 y}{dx^2} \right)^5 + 2 y = 0 \)
5. \( \frac{d^2 y}{dx^2} + y \sin x = 0 \)
6. \( \frac{d^2 y}{dx^2} + x \sin y = 0 \)
7. \( \left( \frac{dy}{dx} \right)^3 = \sqrt{\frac{d^2 y}{dx^2} + 5} \)
1. Show that $y = 4e^{2x} + 2e^{-3x}$ is a solution of the IVP

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0, \quad y(0) = 6, \quad y'(0) = 2.$$ 

2. Is $y = 2e^{2x} + 3e^{3x}$ a solution of the IVP?

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 6y = 0, \quad y(0) = 5, \quad y'(0) = 2.$$ 

3. Show that each of the following IV problems has a unique solution

$$\frac{dy}{dx} = x^2 \sin y, \quad y(1) = -2.$$ 

4. Show that each of the following IV problems has a unique solution

$$\frac{dy}{dx} = \frac{y^2}{x - 2}, \quad y(1) = 0.$$
Show that each function is a solution of the corresponding DE.

1. \( y = x + 3 e^{-x} \) \( \frac{dy}{dx} + y = x + 1 \)
2. \( y = 3 e^{3x} - 2 e^{4x} \) \( \frac{d^2y}{dx^2} - 7 \frac{dy}{dx} + 12 y = 0 \)
3. \( y = 2 e^{3x} + 4 xe^{3x} \) \( \frac{d^2y}{dx^2} - 6 \frac{dy}{dx} + 9 y = 0 \)
4. \( y = 3 e^x + 2x^2 + 6x + 7 \) \( \frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + 2 y = 4x^2 \)
5. \( y = \frac{1}{1+x^2} \) \( (1 + x^2) \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 2 y = 0 \)

Show that \( x^3 + 2xy^2 = 1 \) is a solution of

\[ 2xy \frac{dy}{dx} + x^2 + y^2 = 0. \]

Show that \( 5x^2y^2 - 2x^3y^2 = 1 \) is a solution of

\[ x \frac{dy}{dx} + y = x^3y^3. \]

Show that the following DE has no real solution

\[ \left( \frac{dy}{dx} \right)^2 + y^2 + 1 = 0. \]