Solve the given differential equations in Exercises 1–18.

1. \( \frac{dy}{dx} + \frac{3y}{x} = 6x^3 \)
2. \( x^4 \frac{dy}{dx} + 2xy = 1 \)
3. \( \frac{dy}{dx} + 3y = 3xe^{-y} \)
4. \( \frac{dy}{dx} + 4xy = 8x \)
5. \( \frac{dx}{dt} + \frac{x}{t} = \frac{1}{2t^2} \)
6. \((u^2 + 1) \frac{du}{dx} + 4uu' = 3u \)
7. \( x \frac{dx}{dy} + 2x + 1 = y = x^2 - 1 \)
8. \((x^2 + x - 2) \frac{dy}{dx} + 3(x + 1)y = x - 1 \)
9. \( x \frac{dy}{dx} + (xy + y - 1) dx = 0 \)
10. \( y \frac{dx}{dy} + (xy + x - y) dy = 0 \)
11. \( \frac{dr}{d\theta} + r \tan \theta = \cos \theta \)
12. \( \cos \theta \frac{dr}{d\theta} + (r \sin \theta - \cos \theta) d\theta = 0 \)
13. \((\cos^3 x - y \cos x) dx - (1 + \sin x) dy = 0 \)
14. \( y \sin 2x - \cos x dx + (1 + \sin^2 x) dy = 0 \)
15. \( \frac{dy}{dx} - \frac{y}{x} = \frac{y^2}{x} \)
16. \( \frac{dy}{dx} + y = -2ax^3 \)
17. \( \frac{dy}{dx} + (4y - 8y^2)x dx = 0 \)
18. \( \frac{dx}{dt} + \frac{t + 1}{2t} x = \frac{t + 1}{x} \)

Solve the initial-value problems in Exercises 19–30.

19. \( x \frac{dy}{dx} - 3y = 2x^4, \quad y(2) = 8 \)
20. \( \frac{dy}{dx} + 3x^2 y^2 = x^5, \quad y(0) = 2 \)
21. \( \frac{dy}{dx} = (x^3 + x^2 y) dx + (x^2 + 1) dy = 0, \quad y(0) = 4 \)
22. \( 2x(y + 1) dx - (x^2 + 1) dy = 0, \quad y(1) = -5 \)
23. \( \frac{dx}{dt} + r \tan \theta = \cos^2 \theta, \quad \frac{x}{\theta} = 1 \)
24. \( \frac{dx}{dt} - x = \sin 2t, \quad x(0) = 0 \)

25. \( \frac{dy}{dx} + \frac{y}{x^2} = \frac{x}{y}, \quad y(1) = 2 \)
26. \( x \frac{dy}{dx} + y = (xy)^{y/2}, \quad y(1) = 4 \)
27. \( \frac{dy}{dx} + y = f(x), \quad f(x) = \begin{cases} 2, & 0 \leq x < 1, \\ 0, & x \geq 1 \end{cases}, \quad y(0) = 0 \)
28. \( \frac{dy}{dx} + y = f(x), \quad f(x) = \begin{cases} 5, & 0 \leq x < 10, \\ 1, & x \geq 10 \end{cases}, \quad y(0) = 6 \)
29. \( \frac{dy}{dx} + y = f(x), \quad f(x) = \begin{cases} e^{-x}, & 0 \leq x < 2, \\ e^{-x}, & x \geq 2, \end{cases} \quad y(0) = 1 \)
30. \( (x + 1) \frac{dy}{dx} + y = f(x), \quad f(x) = \begin{cases} 3, & 0 \leq x < 3, \\ 0, & x \geq 3, \end{cases} \quad y(0) = 1/2 \)

31. Consider the equation \( a(y'^2) + by = ke^{-b\theta} \), where \( a, b, \) and \( k \) are positive constants and \( \theta \) is a nonnegative constant. (a) Solve this equation. (b) Show that if \( \lambda = 0 \) every solution approaches \( k/b \) as \( x \to w \), but if \( \lambda > 0 \) every solution approaches \( 0 \) as \( x \to w \).

32. Consider the differential equation
\[ \frac{dy}{dx} + P(x)y = 0. \]
(a) Show that if \( f \) and \( g \) are two solutions of this equation and \( c_1 \) and \( c_2 \) are arbitrary constants, then \( c_1f + c_2g \) is also a solution of this equation.
(b) Extending the result of (a), show that if \( f_1, f_2, \ldots, f_n \) are \( n \) solutions of this equation and \( c_1, c_2, \ldots, c_n \) are \( n \) arbitrary constants, then
\[ \sum_{i=1}^{n} c_i f_i \]
is also a solution of this equation.

33. Consider the differential equation
\[ \frac{dy}{dx} + P(x)y = 0. \]
where \( P \) is continuous on a real interval \( I \).
(a) Show that the function \( f \) such that \( f(x) = 0 \) for all \( x \in I \) is a solution of this equation.
(b) Show that if \( f \) is a solution of (A) such that \( f(x_0) = 0 \) for some \( x_0 \in I \), then \( f(x) = 0 \) for all \( x \in I \).
(c) Show that if \( f \) and \( g \) are two solutions of (A) such that \( f(x_0) = g(x_0) \) for some \( x_0 \in I \), then \( f(x) = g(x) \) for all \( x \in I \).

SECTION 2.2
1. \( (x^2 + 1)y' = c \)
3. \( x^4 + x = c(x - 1) \)
5. \( r \sin \theta = c \)
7. \( (x + 1)(y^2 + 1) = c(x + 2)x \)
9. \( y^2 + ay = cx \)
11. \( y^2 = cx \)
13. \((x^2 + y^2)^{y/2} = x^2 \log cx \)
15. \( x + y = (y / (2x)^{y-1}) \)
16. \( (x^2 + y)^{x/2} = (y^2 + 1)^{x/2} \)
17. \( (2x + y)^{x/2} = (y^2 + 1)^{x/2} \)
19. \( (x^2 + 1)y^x = y^x 

SECTION 2.3
1. \( y = x^2 + ax \)
3. \( y = x^2 + cx^2 \)
5. \( y = x + ax^2 \)
7. \( 3(x^2 + xy) = x^2 - 2x + c \)
9. \( y = y(x^2 + 1) \)
11. \( y = 2 + x^2 + y \)
13. \( (x^2 + 1)x^2 = x^2 + c \)
15. \( y = x^2 - x^2 \)
17. \( y = y(x^2 + 1) \)
19. \( y = x^2 + ax \)
21. \( 2y = x^2 + x^2 \)
23. \( y = x^2 + ax \)
25. \( y = x^2 + ax \)
27. \( \frac{dy}{dx} + \frac{y}{x} = \frac{1}{y} \)
29. \( y = (x + 2 + x^2)^{x/2} \)
31. \( y = 2 + x^2 + y \)

SECTION 2.3. MISCELLANEOUS EXERCISES
1. \( (x^2 + 1)y^x = |y| \)
3. \( x + 1 = c(x + 1) \)